

Mathematical Models for Coupling Rocket Trajectory with Passive Stability

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ABSTRACT

This article develops a unified hierarchy of mathematical models for rocket trajectory and stability. While modern rocket simulators utilize full six degree-of-freedom (6DOF) formulations, these models can be very computationally expensive. They also obscure the true mathematical structure governing the coupling of trajectory and stability. Conversely, simplified rocket models often neglect aerodynamic restoring and damping effects, leading to inaccurate predictions. In this paper a sequence of progressively higher-order dynamical systems is constructed from two to four degrees of freedom. These models are each derived from basic physical principles, verified against the real-world validated rocket simulator RASAeroII. This framework uncovers the minimal set of physical mechanisms required to capture important stability characteristics, including oscillatory modes, damping behavior, and the influence of roll coupling. By tracking the mathematics, the proposed models provide both computational efficiency and a unique physical insight to rocket dynamics. The resulting hierarchy establishes a clear connection between simplified trajectory models and full 6DOF simulations. This offers a practical and understandable foundation for passive stability analysis and rocket simulation.

KEYWORDS

Degrees of Freedom; Trajectory; Apogee; Passive Stability; Caliber; Sounding Rocket; Axisymmetric; Harmonic Oscillator; Force Averaging

INTRODUCTION

Since 1959, NASA's Sounding Rocket Program has provided massive contributions to the national space effort. Designed for sub-orbital missions and surveillance, sounding rockets are a staple in analyzing the physics of rockets within Earth's atmosphere. Sounding rockets, or slender-body rockets, are a class of rockets with axisymmetric, simple bodies¹ - they consist of a nose cone, cylindrical body tube, fins, and commonly a boat tail. Sounding rockets come in a variety of shapes, sizes, and stages, which all largely determine a rocket's stability. If no dynamic control systems are required to achieve a certain caliber, or measure, of stability throughout flight, a rocket is considered to be passively stable. The result of passive stability is of great interest to this article, and directly correlates to the success and versatility in sub-orbital experiments and missions.

Rigid body dynamics in three-dimensional space is completely described by six degree of freedom (6DOF) models, and are applicable to rockets. Such models produced by private space corporations and government agencies each utilize a unique algorithmic model - these models contain many separate nonlinear ODE systems, coupled through code logic.¹ A handful of explicit 6DOF models exist,^{2,3} however these models can be frighteningly complicated, cannot be utilized without proper computing power and time, and thus are outside the scope of this article.

Simplified trajectory models can be found in many places, notably GitHub. However, these models tend to make gross assumptions about tunable aspects such as drag and air density. Clearly, uncoupling trajectory from stability causes inaccuracies that cannot be compensated without data from full simulations. Trajectory and stability in any rigid body must be modeled together. This begs the question - what exactly is the minimal state space required to describe the coupled relationships between trajectory and stability? More specifically, how many degrees of freedom are required to describe the path of a rocket?

In this article, the gap is bridged by determining if higher levels of fidelity yield more meaningful results. First, this article mathematically proves the minimal state-space requirement, a 2DOF model. While this model successfully couples vertical trajectory with pitch attitude, this method lacks agreement with publicly available simulators and stability considerations. Next, a 3DOF model is introduced, which captures true trajectory and stability in the two-dimensional sense. By considering static and dynamic stability, pitch dynamics can be further represented as a second-order harmonic oscillator. Finally, a 4DOF model is constructed, incorporating roll dynamics through Euler's Equations of Motion. This allows the unique point-of-view of the direct results roll has on rocket trajectory and stability. After each system is derived, simulation results follow, thus verifying agreement with proven models. A discussion about this article's findings between the three models follows, ending with conclusions and references.

BACKGROUND

The Black Brant V (five) sounding rocket was developed by Magellan Aerospace in Winnipeg, Canada.¹ It has proven itself as a reliable staple in many sounding rocket programs around the world, including NASA's. Their Sounding Rocket Program Handbook outlines the data and uses for many of their common sounding rockets, including measurements and characteristics of the Black Brant V. Having only one stage, the 'C' variant of this rocket contains four fins and a slightly more powerful motor, when compared to the 'B' variant with three fins. The Black Brant VC's more powerful engine makes it ideal for missions with an apogee, or maximum altitude, from 80 to 260km, carrying payloads from 500 to 1200lbs. This rocket is stackable and comes in many variants, notably the Black Brant IX and XIa, which stack boosters underneath. This allows the aforementioned rockets to achieve much higher altitudes and payload capacities, where usage depends on the mission outline.

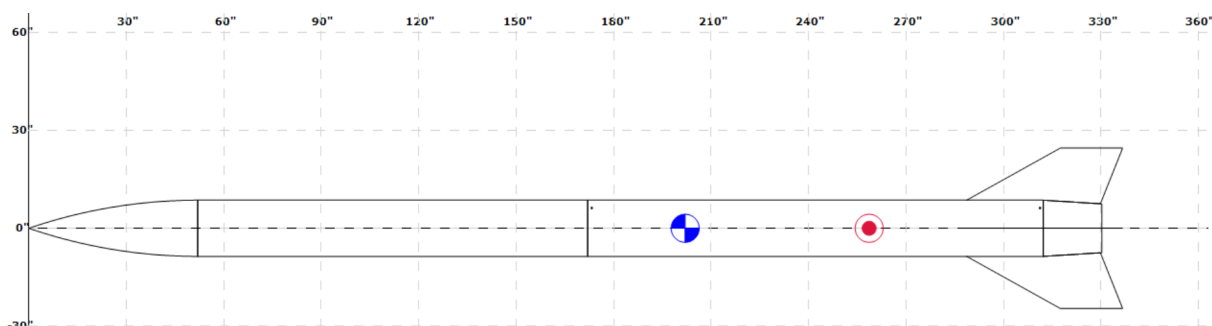


Figure 1. The Black Brant VC Sounding Rocket, modeled inside RASAeroII.

Using RASAeroII, which is a 3DOF rocket simulator,⁴ the Black Brant VC was modeled in **Figure 1**. RASAeroII boasts an average accuracy within 3.47% of real world results for apogees under 300000 feet, and a general average real world accuracy of five percent for apogees above 300000 ft.⁴ Verification of the models produced in this article will be completed based on the agreement between RASAeroII, as gathering real world data from sounding rockets is outside the scope and ability of this article.

When discussing rocket stability, the most important measure comes down to the *stability margin*, defined as the relative distance between the centers of gravity and pressure. Center of pressure should live near the fins of a stable rocket, where the average surface area resides, and moves with mach speed. Under subsonic conditions, when mach speed is less than 0.8, the center of pressure barely moves. During the transonic region between mach 0.8

and 1.2, center of pressure spikes aft mostly due to wave drag. Finally, at supersonic speeds above mach 1.2, the center of pressure moves forwards. On the other hand, center of gravity lives at the balance point of a rocket, and moves forwards depending on how quickly fuel mass depletes.⁵ Once fuel mass depletes, it can be expected that the center of gravity does not change for the duration of flight.

Definition 1. Let x_{CG} and x_{CP} be the positions of center of gravity and pressure, respectively. Additionally, let D be the diameter of the rocket's body tube. Then stability margin SM is calculated by:⁶

$$SM = \frac{x_{CP} - x_{CG}}{D}, \quad \text{Equation 1.}$$

where a stable rocket has a **positive** stability margin. The larger this value, the more a rocket will counter perturbing forces with restoring moments.

The rate of change of restoring moments produced by a rocket, about pitch, is C_{m_α} . This static stability term is the derivative of the moment coefficient C_m w.r.t. the angle of attack α .

Definition 2. Let C_{N_α} represent the rate of change of normal forces experienced by the rocket in the pitch direction, calculated using the Rogers Modified Barrowman Method.⁴ Then the term C_{m_α} is best estimated using the stability margin SM :²

$$C_{m_\alpha} = -SM C_{N_\alpha}. \quad \text{Equation 2.}$$

The rate of change of damping about pitch arises from aerodynamic forces induced with rotational motion.⁵ This term represents the rate at which a rocket will increase or decrease oscillations to reach a steady state. C_{m_α} is referred to as the **pitch stability derivative**, and C_{m_q} as the **pitch damping derivative**.

Definition 3. Let C_{m_q} be the rate at which oscillations are damped due to restoring moments. Then the best estimation of C_{m_q} can be calculated as:²

$$C_{m_q} = -2C_{N_\alpha} SM, \quad \text{Equation 3.}$$

which assumes aerodynamic forces are concentrated around center of pressure x_{CP} .

For rocket stability, both derivatives C_{m_α} and C_{m_q} must be **negative** for stable flight to occur. Else, the rocket tumbles.⁵ Air density within Earth's atmosphere ρ decreases exponentially with altitude x , described below using the exponential density profile of Earth's atmosphere:⁵

$$\rho = \rho_0 e^{-\frac{x}{H}}. \quad \text{Equation 4.}$$

Using atmospheric ceiling height $H = 850000$ meters, a plot is presented over altitude x , and air density with altitude can be viewed in **Figure 2** as a logarithmic plot to better show the exponential nature of **Equation 4**. Gravitational acceleration experienced by an object in Earth's atmosphere is determined by²

$$g = g_0 \left(\frac{r_e}{r_e + x} \right)^2, \quad \text{Equation 5.}$$

where g_0 is gravity at sea level and r_e is the radius of Earth, in meters. Viewing the plot in **Figure 3**, it's clear to see how gravitational acceleration decreases almost linearly when leaving Earth's surface.⁵ Additionally, the total mass of a rocket is calculated by the sum of constant dry mass m_{dry} and current liquid fuel mass m_f ,

$$m = m_{dry} + m_f. \quad \text{Equation 6.}$$

The thrust curve T describes thrust force, in Newtons, produced over time. This is where maximum thrust and action time are described. The thrust interpolant for the Black Brant VC sustainer is plotted below in **Figure 4**, derived from data found in NASA's Sounding Rocket Program Handbook,¹ and used similarly in both RASAeroII and MATLAB simulations to enforce the verification of model agreement.

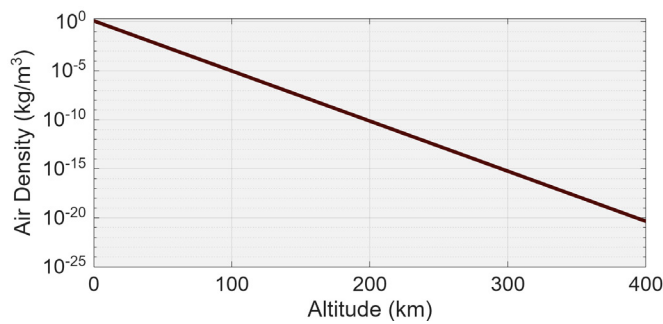


Figure 2. Air density over altitude, in kilometers (logarithmic scale).

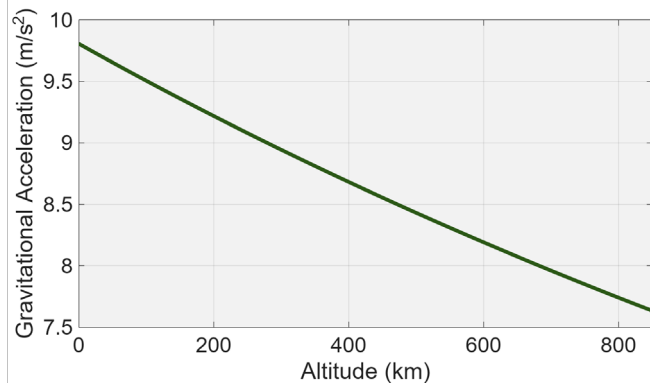


Figure 3. Gravity over altitude, in kilometers.

The temperature model T_a describes the change in temperature, in Kelvin, as altitude increases through the layers of Earth's atmosphere.⁷ Due to the scope of this article's models, the piecewise temperature model below accounts for the first $H = 400$ kilometers of Earth's atmosphere from sea level to the edge of the thermosphere, constructed using measurements of temperatures throughout the layers of Earth's atmosphere from the National Oceanic and Atmospheric Administration (NOAA):

$$T_a = \begin{cases} \frac{26.65 - T_0}{11000}x + T_0 & \text{if } 0 \leq x < 11000 \\ 216.65 & \text{if } 11000 \leq x < 20000 \\ \frac{53.35}{30000}x + 181.05 & \text{if } 20000 \leq x < 50000 \\ 270.13 & \text{if } 50000 \leq x < 53000 \\ -\frac{97}{36000}x + 404.1 & \text{if } 53000 \leq x < 86000 \\ 173 + 827(1 - \exp^{-\frac{x-86000}{198500}}) & \text{if } x \geq 86000 \end{cases}, \quad \text{Equation 7.}$$

where x is in meters, and T_0 represents temperature at sea level in Kelvin, varying based on season and location. Plotting the temperature model in **Figure 5** helps to understand how temperature is proportional to density until solar absorption occurs, increasing temperatures above 86 kilometers rapidly from sea level.

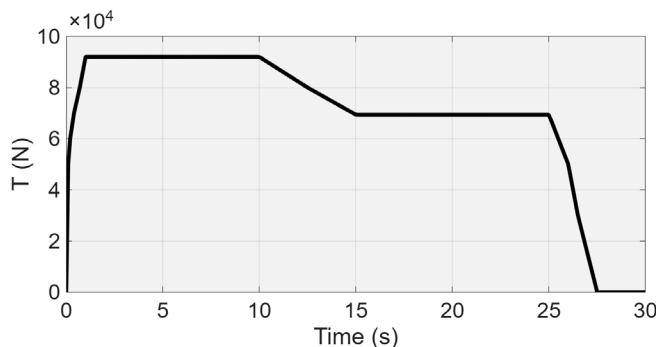


Figure 4. Thrust profile of the Black Brant VC sustainer, in Newtons.

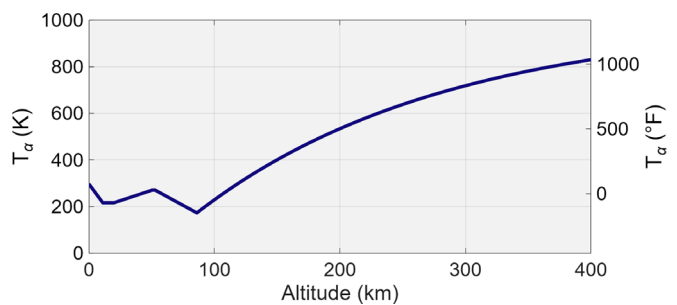


Figure 5. Temperature over altitude, in kilometers.

To find mach speed of a rocket traveling through the layers of Earth's atmosphere, the speed of sound a is first calculated by the equation below as¹

$$a = \sqrt{\gamma_a R T_a}, \quad \text{Equation 8.}$$

where $\gamma_a = 1.4$ is the ratio of specific heats, and $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ is the specific gas constant. Thus, the speed of sound only fluctuates with temperature T_a , determined by altitude x as seen from **Equation 7**. Mach number v_m

can now be defined by dividing positive velocity by the speed of sound:¹

$$v_m = \frac{|v|}{a}. \quad \text{Equation 9.}$$

Pitch inertia I_y , which depends on current fuel mass m_f , is best estimated for sounding rockets like the Black Brant VC by adding the inertial contributions of the body and liquid fuel. Inertia represents an object's resistance to rotation about an axis. The equation below estimates the inertia for a thin-walled cylinder:⁸

$$I_y = \frac{1}{12}m(3R^2 + L_t^2) + m_f(x_{aft} - x_{CG})^2, \quad \text{Equation 10.}$$

where R is the body radius, L_t is total length of the rocket, and x_{aft} is the distance between the nosecone and center of the fuel cell, all in meters. The rate of change of fuel mass m_f (kilograms), known as fuel burn rate m'_f ($\frac{kg}{s}$), is given by¹

$$m'_f = -\frac{T}{g_0 I_{sp}}, \quad \text{Equation 11.}$$

where I_{sp} is the specific impulse of the rocket motor. When I_{sp} is multiplied by sea level gravity g_0 , muzzle velocity is produced.⁸ The specific impulse of a motor is the measure of seconds in which one unit of thrust can be produced from one unit of fuel. This reflects the efficiency in which fuel is converted to thrust for a motor, with a larger specific impulse implying a greater fuel efficiency.

TWO DEGREES OF FREEDOM - METHODS AND PROCEDURES

In theory, the simplest way to couple rocket trajectory and stability would be a 2DOF model, where only one axis each describing each are considered. The three axes of a rocket, in reference to its body, can be viewed in **Figure 6**. Each axis can describe translation along with rotation, and thus objects in the 3D sense have six degrees of freedom. Vertical translation, or traveling along axis x^B , can produce results like altitude and apogee, whereas rotation about axis y^B describes pitch attitude for stability.

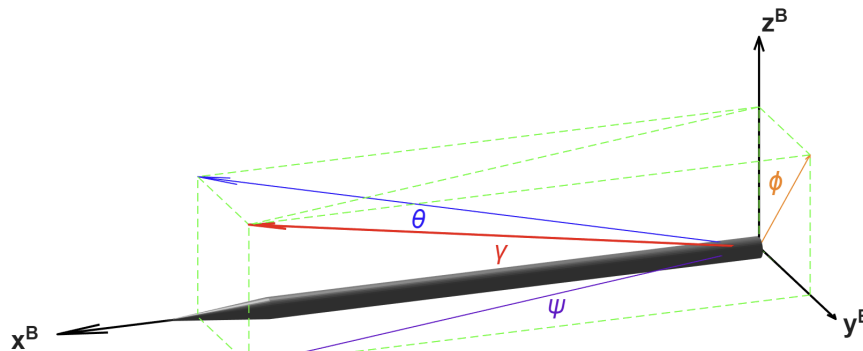


Figure 6. Rocket body reference axes.

From **Figure 6**, attitudes ϕ , θ , and ψ represent roll, pitch, and yaw, describing radians of rotation about axes x^B , y^B , and z^B , respectively. Flight path angle γ represents the inverse tangent of the ratio of velocities in the vertical / horizontal plane, expanded upon in the 3DOF section. This vector is the resulting direction of where the rocket is moving, not necessarily where the nose is pointing. Finally, when the rocket is rotated 90° clockwise about axis y^B , translation along x^B becomes altitude, y^B is distance, and z^B as sidewalk.

TRAJECTORY - 2DOF

Consider translation along the x^B axis - when pitch attitude $\theta = \frac{\pi}{2}$, x^B is vertical, and so is the rocket. From Newton's 2nd Law of Motion, the sum of accelerative rates⁸ can be calculated to define the acceleration a rocket experiences. Taking the time derivative of displacement x gives velocity v , and once again yields acceleration v' .

These values must be simultaneously ‘vertical’ and ‘overall’ due to the limitation of this model, which has only one translational axis:

$$\begin{aligned} x' &= v, \\ v' &= \frac{T}{m} - g - \frac{F_D}{m}. \end{aligned} \quad \text{Equation 12.}$$

From **Equation 12**, F_D is the force experienced by the rocket due to drag.⁵ Drag force F_D is calculated below as

$$F_D = \bar{q} S C_D, \quad \text{Equation 13.}$$

where dynamic pressure $\bar{q} = \frac{1}{2} \rho v^2$, reference area $S = \pi r^2$, and drag coefficient C_D are considered. The drag coefficient changes with mach speed v_m and angle of attack α , requiring an extensive model of its own for realistic calculations. The types of drag considered include base, wave, pressure, skin friction, and drag due to angle of attack, with slight differences between engine on and off conditions.

STABILITY - 2DOF

Now, consider rotation about axis y^B - in the strictly vertical model, it becomes clear that the directions in which pitch attitude (θ) and flight path angle (γ) vectors follow must live along axis x^B . This leads to an intuitive but important result.

Lemma 1. *If a rigid body model is limited to one axis each of translation and rotation, then attitude must remain constant.*

Proof. Consider a 2DOF rocket model, tracking vertical translation and transverse rotation. Angle of attack is defined as

$$\alpha = \theta - \gamma, \quad \text{Equation 14.}$$

where θ is pitch attitude and γ is flight path angle, in radians. Because only vertical travel is considered, no deviation from this axis can be tracked. Intuitively, $\alpha = 0$ always - it follows this limitation holds for the other translational directions, such as distance or sidewalk with yaw attitude. Therefore, every rigid body model with only one axis each of translation and rotation must have attitude remaining constant. $\square$

Within the 2DOF vertical flight model, $\theta = \gamma$ and thus $\alpha = 0$. This result alone suggests the need for an additional translational degree of freedom. Nonetheless, this article acknowledges the mathematical possibility of analysis, and assigns the rest of the state variables needed to see what results can be obtained.

Pitch angular acceleration q' is calculated by dividing the aerodynamic pitching moment M by pitch inertia I_y .⁵, consisting of both static and dynamic terms, derived from Euler’s Equations of Motion:⁸

$$M = \bar{q} S \ell (C_{m_\alpha} \alpha + C_{m_q} \frac{\ell}{2v} q). \quad \text{Equation 15.}$$

Reference length ℓ is the distance, in meters, from center of gravity to center of pressure - when $C_{m_\alpha} < 0$, aerodynamic moment *opposes* angle of attack. This stabilizes the rocket along flight path γ . Additionally, when $C_{m_q} < 0$, the rocket experiences a resistance to angular acceleration, defining the rates at which aerodynamic moments dampen oscillations caused by these restoring moments.

It’s imperative to understand that the sign of SM , from **Equation 1**, will subsequently determine the signs of both C_{m_α} and C_{m_q} in this model. When the stability margin is *positive*, the rocket will experience correct perturbing forces with an ever-increasing amount of torque, causing instability and tumbling. Additionally, if these rates are negative, but too small, the rocket will fail to restore stability about pitch quickly enough for reliable flight.

The stability subsystem is now:

$$\begin{aligned}\theta' &= q, \\ q' &= \frac{M}{I_y},\end{aligned}\tag{Equation 16.}$$

and by **Lemma 1**, $\alpha = 0$, thus only damping effects remain, removing stiffness from **Equation 16** entirely. All together, a model is constructed describing the coupling between vertical trajectory and pitch damping in two degrees of freedom.

$$\begin{aligned}x' &= v, \\ v' &= \frac{T}{m} - g - \frac{F_D}{m}, \\ \theta' &= q, \\ q' &= \frac{\bar{q}S\ell^2}{4vI_y}C_{m_q}q.\end{aligned}\tag{Equation 17.}$$

Equation 17 mathematically couples trajectory into stability, however no freedom exists for pitch attitude θ , flight path angle γ , or angle of attack α . Still, results agreeable with RASAeroII can be shown.

RESULTS - 2DOF

Utilizing RASAeroII, results of the Black Brant VC were obtained under purely vertical flight, equivalent to the fidelity of a 2DOF model. Most sounding rockets employ yo-yo despin, parachute, and drogue phases, however these consideration have been removed for the simplicity of testing the mathematical models' agreement with RASAeroII. It is the scope of this article to verify the mathematical agreement between the two models, as opposed to measuring accuracy to real world data which is unobtainable for this rocket specifically.

Simulation results of the Black Brant VC from **Equation 17** is now shown below, overlaid by the data obtained from the RASAeroII simulation. The MATLAB 2DOF model underestimates apogee by 0.9%, mostly due to a lack of stiffness in stability, while maximum and minimum speeds are within 0.3%. Although the considerations of the 2DOF model are too low to be physically reliable, they still agree with RASAeroII's estimations within 1%.

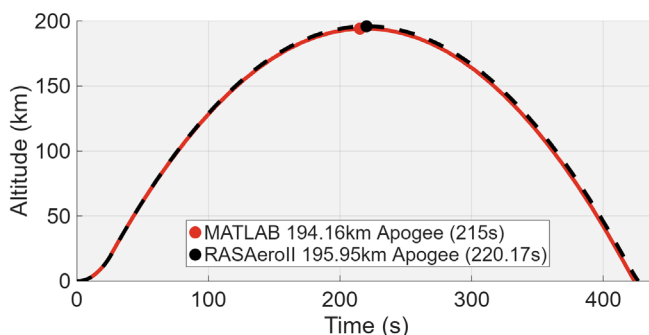


Figure 7. Altitude over time of the Black Brant VC compared between RASAeroII (dashed black line) and the MATLAB 2DOF simulator (solid red line).

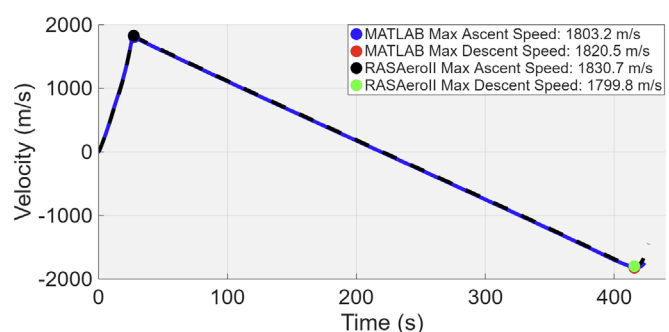


Figure 8. Vertical velocity over time of the Black Brant VC compared between RASAeroII (dashed black line) and the MATLAB 2DOF simulator (solid blue line).

THREE DEGREES OF FREEDOM - METHODS AND PROCEDURES

In order to properly capture stability, it's clear from **Lemma 1** that α must be nonzero. More specifically, pitch attitude θ and flight path angle γ must be independent of each other,⁸ and free to change during flight. It is here this article takes a moment to clarify the assumption of an axisymmetric rocket - when viewed from the nosecone along axis x^B , the rocket may not be perfectly symmetric in design, and geometric centers remain along x^B always.¹ Based on **Equation 14** from **Lemma 1**, a third degree of freedom is necessary to describe α this way.⁸ By

choosing to model travel along axis y^B , flight path angle γ can be defined as the $\arctan()$ of the ratio of velocities

$$\gamma = \arctan\left(\frac{v_x}{v_y}\right), \quad -\pi \leq \gamma \leq \pi \quad \text{Equation 18.}$$

where v_x is velocity in the vertical sense along x^B , and v_y horizontally along y^B . The ratio of these velocities yields drift in a two-dimensional plane, allowing for γ to reflect resulting flight path angle.⁵ Taking advantage of this, the rates of change of velocity can be constructed by finding the magnitude of vertical and horizontal velocity vectors.⁵ The system below describes vertical translation now,

$$\begin{aligned} x' &= v_x, \\ v_x' &= \frac{T \sin \theta}{m} - g - \frac{v_x F_D}{vm}. \end{aligned} \quad \text{Equation 19.}$$

and horizontal translation can be modeled similarly:

$$\begin{aligned} y' &= v_y, \\ v_y' &= \frac{T \cos \theta}{m} - \frac{v_y F_D}{vm}. \end{aligned} \quad \text{Equation 20.}$$

Overall velocity v is now⁵

$$v = \sqrt{v_x^2 + v_y^2}, \quad \text{Equation 21.}$$

and the pitch stability subsystem remains the same as **Equation 16** with no additional considerations. The 3DOF model becomes:

$$\begin{aligned} x' &= v_x, \\ v_x' &= \frac{T \sin \theta}{m} - g - \frac{v_x F_D}{vm}, \\ y' &= v_y, \\ v_y' &= \frac{T \cos \theta}{m} - \frac{v_y F_D}{vm}, \\ \theta' &= q, \\ q' &= \frac{M}{I_y}. \end{aligned} \quad \text{Equation 22.}$$

THREE DEGREES OF FREEDOM - RESULTS

A 3DOF rocket simulator can now be implemented in MATLAB. This time, the versatility of **Equation 22** is highlighted by introducing *initial launch angles*. The Black Brant VC is evaluated in the figures below when launched at zero, five, ten, and fifteen degrees from vertical, displaying how choosing such a condition effects trajectory results. RASAeroII continues to be the comparative benchmark for this fidelity.

Under vertical launch, the MATLAB simulator estimates apogee with five orders of agreement to RASAeroII's estimation. As launch angle perturbs from zero, the MATLAB simulator retains at least two orders of agreement for apogee, distance, and speed estimations, highlighted in **Figures 9 to 12**.

With these results, a theorem is introduced clarifying the distinction between two and three DOF stability subsystems, and the difference this makes for analyzing passive stability of rockets.

Theorem 2. *A rocket dynamics model exhibits a genuine coupling between trajectory and stability in the 2D sense iff a minimum of three degrees of freedom are considered.*

Proof. A proof by contrapositive follows, first showing $\neg B \Rightarrow \neg A$, then concludes by showing $B \Rightarrow A$:

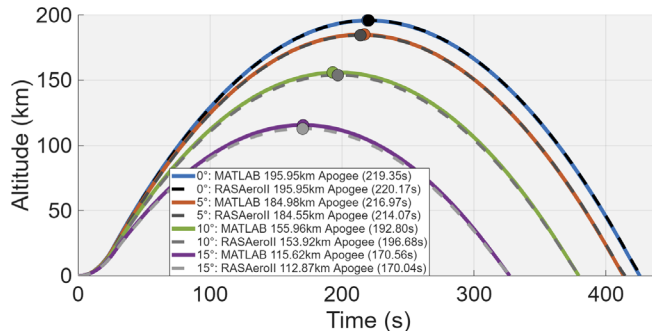


Figure 9. Altitudes over times of the Black Brant VC compared between RASAeroII (dashed grayscale lines) and the MATLAB 3DOF simulator (solid color lines).

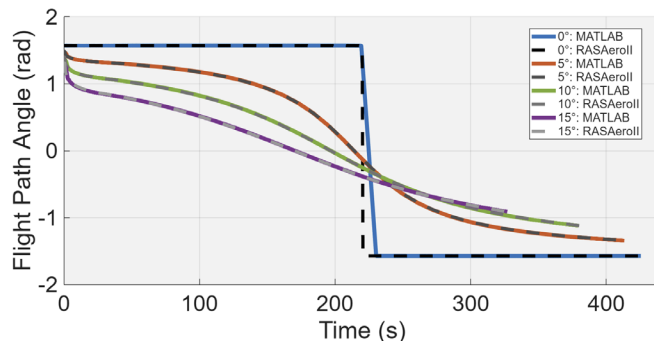


Figure 11. Flight path angles over times of the Black Brant VC compared between RASAeroII (dashed grayscale lines) and the MATLAB 3DOF simulator (solid color lines).

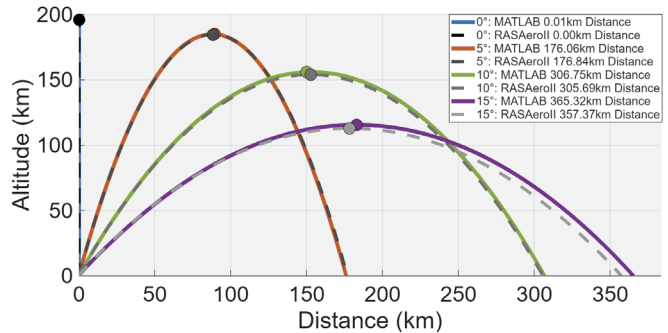


Figure 10. Altitudes over distances of the Black Brant VC compared between RASAeroII (dashed grayscale lines) and the MATLAB 3DOF simulator (solid color lines).

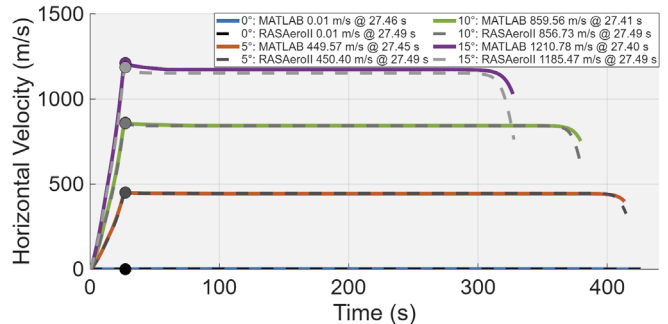


Figure 12. Horizontal velocities over times of the Black Brant VC compared between RASAeroII (dashed grayscale lines) and the MATLAB 3DOF simulator (solid color lines).

Let a rocket dynamics model exhibit a true coupling between trajectory and stability. This means the position of the rocket in a 2D plane is free to move in two different directions, and the attitude in this plane can both affect and be affected by the trajectory of the rocket.

Assume, to the contrary, this rocket dynamics model has fewer than three degrees of freedom. Then two cases follow:

Case 1: One translational and one rotational degree of freedom are considered. By **Lemma 1**, the attitude must remain constant.

Case 2: Two translational degrees of freedom are considered. Then the model is purely translational, with no stability coupling.

In both cases, a contradiction is reached - neither of these cases properly couple trajectory and stability. Thus a rocket dynamics model with fewer than three degrees of freedom cannot exhibit a genuine coupling of the two.

Conversely, assume a model has three degrees of freedom in the two-dimensional sense, consisting of two translational and one rotational. Then attitude affects the trajectory in this plane by determining the direction of acceleration. Additionally, trajectory affects the aerodynamic moments governing rotational acceleration. This is exactly what produces a genuine coupling between trajectory and stability in the 2D sense, so a rocket dynamics model must contain at least three degrees of freedom to describe this coupling.

Therefore, a rocket dynamics model exhibits a genuine coupling between trajectory and stability in the 2D sense if and only if a minimum of three degrees of freedom are considered. $\square$

Expanding upon **Theorem 2**, consider the 3DOF stability subsystem, from **Equation 16**:

$$\begin{aligned}\theta' &= q, \\ q' &= \frac{\rho v^2 S \ell}{2I_y} C_{m_\alpha} (\theta - \gamma) + \frac{\rho v S \ell^2}{4I_y} C_{m_q} q,\end{aligned}\tag{Equation 23.}$$

and define terms

$$k = -\frac{\rho v^2 S \ell}{2I_y} C_{m_\alpha},\tag{Equation 24.}$$

$$c = -\frac{\rho v S \ell^2}{4I_y} C_{m_q},\tag{Equation 25.}$$

where k is denoted stiffness and c as damping⁸. Then **Equation 23** gives the second-order harmonic oscillator⁸

$$\theta'' + c\theta' + k(\theta - \gamma) = 0.\tag{Equation 26.}$$

Natural frequency ω_n and damping ratio ζ can now be derived from the oscillator. The damped frequency ω_d represents the decay of oscillation frequencies experienced by the rocket:

$$\omega_n = \sqrt{k}, \quad \zeta = \frac{c}{2\omega_n}, \quad \omega_d = \omega_n \sqrt{1 - \zeta^2}.$$

Taking the partials of **Equation 23**, the Jacobian is evaluated as:

$$J = \begin{pmatrix} 0 & 1 \\ -k & -c \end{pmatrix},\tag{Equation 27.}$$

with eigenvalues

$$\begin{aligned}\lambda_{1,2} &= -\frac{c \pm \sqrt{c^2 - 4k}}{2}, \\ &\equiv -\zeta\omega_n \pm i\omega_d.\end{aligned}\tag{Equation 28.}$$

The real part of the eigenvalues from **Equation 28** is:

$$\text{Re}(\lambda_{1,2}) = -\frac{c}{2} = -\zeta\omega_n,\tag{Equation 29.}$$

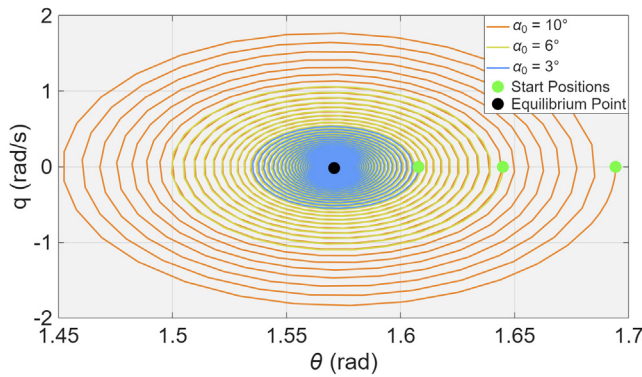
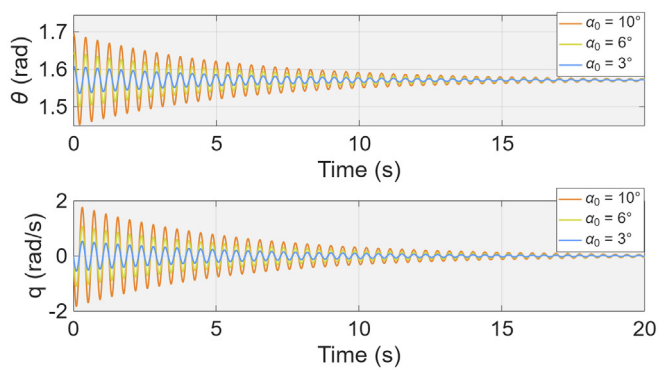
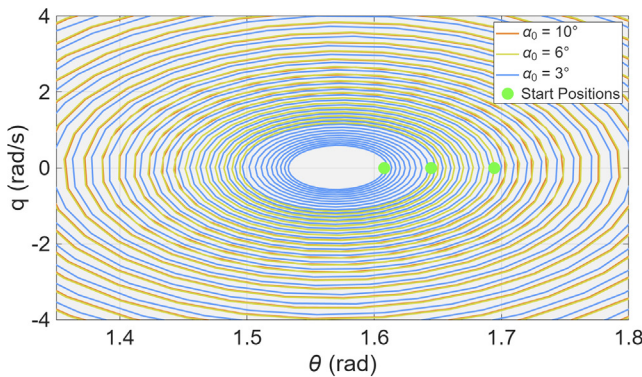
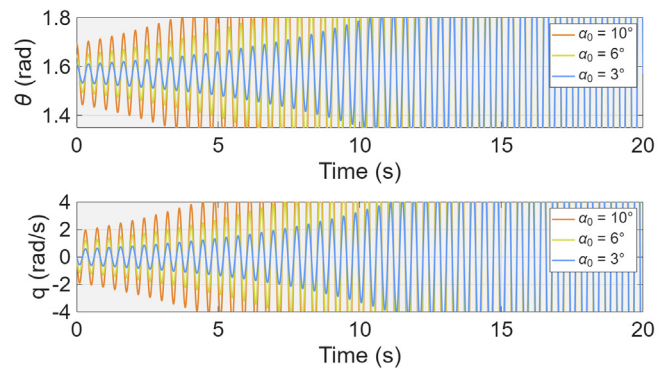
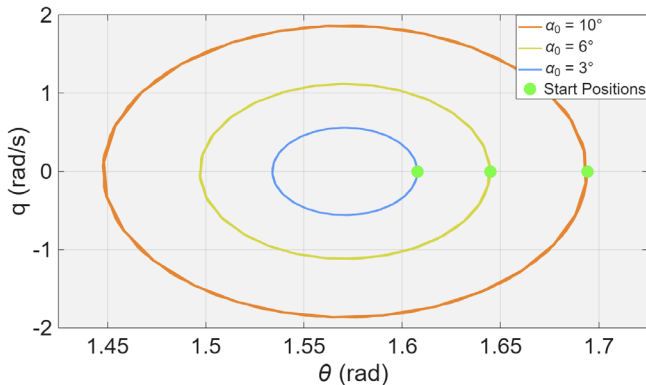
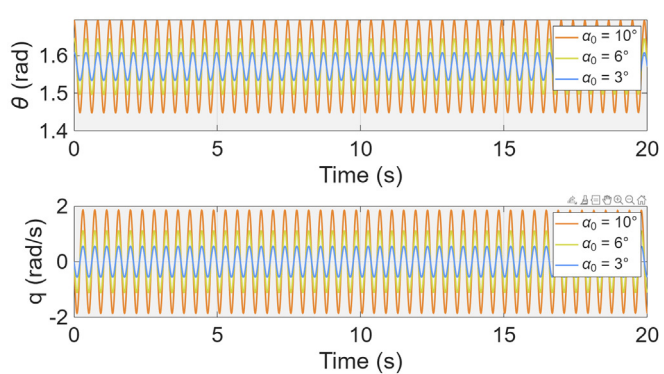
and because of the expansion of damping term c from **Equation 25**, the eigenvalues have a negative real part only if pitch damping derivative $C_{m_q} < 0$, giving both $c > 0$ and $\zeta > 0$. Because of the velocity term being squared in the numerator of k but not in c , the discriminant of the eigenvalues is guaranteed to always be negative. Thus, when $c > 0$, a *stable* spiral sink is formed. If pitch damping derivative $c = 0$, oscillations are simply undamped and form a limit cycle. Finally, when $c < 0$, *unstable* flight occurs in the shape of a spiral source. A very simple corollary follows:

Corollary 3. *Rocket stability is determined by the stability margin.*

Proof. When the stability margin is positive, the damping derivative is negative, and damping term c is positive. When $c > 0$, the eigenvalues from **Equation 28** are negative, and the rocket is stable. It follows that when the stability margin is negative, the rocket is unstable due to the eigenvalues being positive. Thus the stability of a rocket is determined by the stability margin. $\square$

Using values at $t = 15$ seconds from the 3DOF MATLAB simulation, phase portraits can be constructed at different perturbations of angle of attack as seen below. These figures represent the pitch stabilization of the Black Brant VC over a local timespan. Because fifteen seconds into launch is near the moment of largest dynamic pressure, these plots represent the moment of greatest instability for this rocket specifically. The solution timeplots for $\theta(t)$ and $q(t)$ are shown comparatively, considering these angles of attacks in **Figures 13 to 18**.

To demonstrate the importance of damping term c in determining the sign of the real part of the eigenvalues, the phase portraits and timeplots below display how pitch stability changes when $c < 0$ and $c = 0$, creating unstable

Figure 13. Phase portraits when $c > 0$.Figure 14. Timeplots when $c > 0$.Figure 15. Phase portraits when $c < 0$.Figure 16. Timeplots when $c < 0$.Figure 17. Phase portraits when $c = 0$.Figure 18. Timeplots when $c = 0$.

FOUR DEGREES OF FREEDOM - METHODS AND PROCEDURES

Moving into new territory, a second rotational degree of freedom about either roll (axis x^B) or yaw (axis z^B) can be added, resulting in a 4DOF model. First, Euler's Equations of Motion for rigid bodies must be considered,⁸ one of the many ODE systems involved in modern 6DOF rocket simulators. Euler's Equations of Motion are defined about axes x^B , y^B , and z^B as follows:

$$\begin{aligned} L &= I_x p' + (I_z - I_y)qr, \\ M &= I_y q' + (I_x - I_z)pr, \\ N &= I_z r' + (I_y - I_x)pq. \end{aligned} \quad \text{Equation 30.}$$

Here, angular accelerations p' , q' , and r' are referenced about roll, pitch, and yaw, respectively. Angular rates p , q , and r are referenced in the same manner, along with aerodynamic moments about roll L , pitch M , and yaw N .⁸

Principal inertial tensors I_x , I_y , and I_z all correspond to their respective axes (roll, pitch, and yaw), and one can see the derivation for aerodynamic pitching moment M and pitch inertia I_y from **Equations 15 and 10** in the previous models. Recall the assumption made earlier about axisymmetric rockets - a theorem is provided below about the unique reduction of Euler's Equations of Motion.⁸

Theorem 4. *Let B be a rigid rocket body whose mass distribution is axisymmetric about its longitudinal axis x^B . Then the following properties hold simultaneously:*

- Roll angular acceleration is dynamically uncoupled from both pitch and yaw angular accelerations.
- Roll rate is the sole angular rate coupling term between pitch and yaw dynamics.

Proof. Let a rocket's body B be axisymmetric about axis x^B , so $I_y = I_z$. Then **Equation 30** can be reduced and rewritten as a system of angular acceleration about roll, pitch and yaw:

$$\begin{aligned} p' &= \frac{L}{I_x}, \\ q' &= \frac{M + (I_y - I_x)pr}{I_y}, \\ r' &= \frac{N - (I_y - I_x)pq}{I_y}, \end{aligned} \quad \text{Equation 31.}$$

where roll angular acceleration p' is no longer influenced by pitch or yaw dynamics, and roll rate p is the sole angular rate coupling pitch and yaw dynamics.⁸ □

Because of this result, the model can be extended to four degrees of freedom in the following unique manner:

$$\begin{aligned} x' &= v_x, \\ v'_x &= \frac{T \sin \theta}{m} - g - \frac{v_x F_D}{vm}, \\ y' &= v_y, \\ v'_y &= \frac{T \cos \theta}{m} - \frac{v_y F_D}{vm}, \\ \theta' &= q, \\ q' &= \frac{M + (I_y - I_x)pr}{I_y}, \\ \psi' &= r, \\ r' &= \frac{N - (I_y - I_x)pq}{I_y}, \end{aligned} \quad \text{Equation 32.}$$

where ψ is yaw attitude, r is yaw rate, and r' is yaw angular acceleration from Euler's Equations of Motion. This way, nose coning motion can be considered between attitudes θ (pitch) and ψ (yaw), and roll rate p becomes a tunable parameter, directly influencing the transfer of energy between these angular systems.

When it comes to passive stabilization in the real world, maintaining a roll rate p is essential to capitalizing on stability by influencing natural frequency ω_n and damping ratio ζ . Events like shooting a bullet and throwing a football are great examples of axisymmetric objects that require spin stabilization for desired flight. This gyroscopic stiffness effect is what allows smaller flying bodies that are too marginally stable on their own to be artificially stabilized with roll.

In order to maintain stability in a variety of conditions, the Black Brant VC has an average prescribed roll rate of 3.5 cycles per second,¹ which is perfect for ideal force averaging⁸ without the risk of structural damage due

to over damping. First, this article considers the new, four equation stability subsystem from **Equation 32** with hopes of seeing what effects roll has on passive stability through phase plane analysis. Expanded, the following subsystem is obtained:

$$\begin{aligned}\theta' &= q, \\ q' &= \frac{\bar{q}S\ell}{I_y}C_{m_\alpha}\alpha + \frac{\bar{q}S\ell^2}{2vI_y}C_{m_q}q + \frac{(I_y - I_x)p}{I_y}r, \\ \psi' &= r, \\ r' &= \frac{\bar{q}S\ell}{I_y}C_{n_\beta}\psi + \frac{\bar{q}S\ell^2}{2vI_y}C_{n_r}r - \frac{(I_y - I_x)p}{I_y}q,\end{aligned}\tag{Equation 33.}$$

and only because of the axisymmetric assumption, it may be assumed that stability derivatives $C_{m_\alpha} \equiv C_{n_\beta}$, where C_{n_β} is the rate of change of restoring moments about yaw axis z^B (from N). Similarly, damping derivatives $C_{m_q} \equiv C_{n_r}$, where C_{n_r} is the rate of change of damping about the yaw axis z^B (from N).⁵ Thus the claim that $M \equiv N$ for this axisymmetric rocket holds. Recall the previous stiffness and damping terms⁸ k and c from **Equations 24 and 25**:

$$\begin{aligned}\theta'' + c\theta' + k(\theta - \gamma) - \Omega\psi' &= 0, \\ \psi'' + c\psi' + k\psi + \Omega\theta' &= 0.\end{aligned}\tag{Equation 34.}$$

Define complex variable $z = (\theta - \gamma) + i\psi$, so $z' = q + ir$ and $z'' = q' + ir'$. Solving for z'' produces a single, second-order complex harmonic oscillator, representing the coupling between pitch and yaw from **Equation 34**.

$$z'' + (c + i\Omega)z' + kz = 0\tag{Equation 35.}$$

Here, Ω in **Equation 35** is the term $\frac{(I_y - I_x)p}{I_y}$, responsible for coupling pitch and yaw. When roll rate $p = 0$, $\Omega = 0$, and the pitch and yaw stability subsystems become uncoupled and independent. It becomes clear now exactly how energy is transferred between pitch and yaw - in addition to coupling these rotational axes, roll rate coupling term Ω provides a deeper result for natural frequency ω_n and damping ratio ζ .

$$\omega_n = \sqrt{k}, \quad \zeta = \frac{c + i\Omega}{2\omega_n}, \quad \omega_d = \omega_n \sqrt{1 - \zeta^2}.$$

Interestingly enough, roll rate does not effect a rocket's static stiffness, as this is a geometric property primarily governed by stability margin SM from **Equation 1**. Rather, roll is responsible for coupling pitch and yaw dynamics, modifying dynamic stability instead. What must be understood about roll is that this coupling can alter the natural frequency ω_n and damping ratio ζ , but not necessarily in a strictly 'increasing' or 'decreasing' manner.

Roll is best understood as a mechanism for averaging the aerodynamic and structural asymmetries of a rocket, and thus when a rocket is ideally axisymmetric, like the Black Brant VC, this effect is minimal. However, in practical, real world systems where small imperfections always exist, roll can reduce the influence these asymmetries contribute to instability. As a result, a rocket that rolls has the unique ability to reduce localized stresses on the rocket body frame by distributing these aerodynamic loads more evenly about the two axes during flight, preventing events like bending frames and breaking fins.

In some cases, introducing roll can help mitigate unwanted dynamic effects, such as those encountered near maximum dynamic pressure or during transonic flight. By preventing these 'directional biases' in pitch and yaw, roll promotes a more balanced response to perturbations, though it does not guarantee increased damping nor overall stability. The amount of perturbing forces can neither be increased nor decreased, only redistributed.

FOUR DEGREES OF FREEDOM - RESULTS

Equation 32 was coded in MATLAB with the goal of viewing the real world effects spin has on the Black Brant VC specifically. With a launch angle of five degrees from vertical, the following plots compare results obtained

with and without roll, implemented in the simplest fashion by using a linear interpolant to increase roll from zero to $p = 3.5$ cycles per second by fuel burnout, then maintained for the duration of flight. It's important to note this is not realistic, but rather the simplest way to view the effects rolling the Black Brant VC has on previously calculated results.

Additionally, it follows from **Theorem 4** that when roll rate $p = 0$, the pitch angular acceleration equation simplifies to $q' = \frac{M}{I_y}$, and thus the 4DOF model simplifies to the 3DOF model covered in the previous section. For this reason, the plots below are not compared with data from the RASAeroII simulator due to redundancy in results, and the fact that RASAeroII does not support such rolling conditions as a 3DOF simulator.

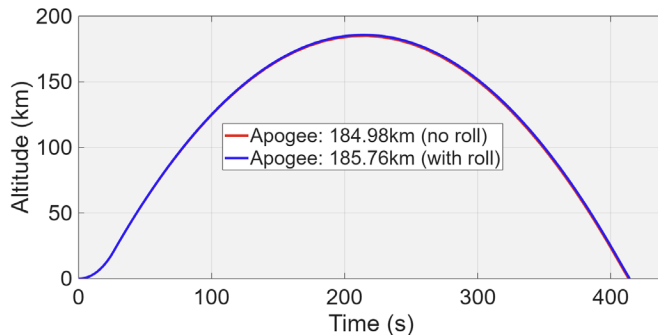


Figure 19. MATLAB 4DOF Simulator - altitude over time, comparing rolling and non-rolling conditions.

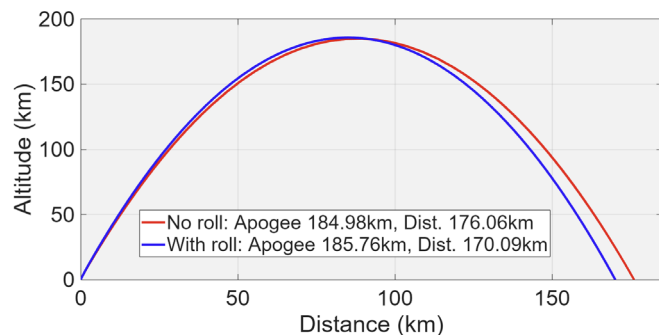


Figure 20. MATLAB 4DOF Simulator - altitude over distance, comparing rolling and non-rolling conditions.

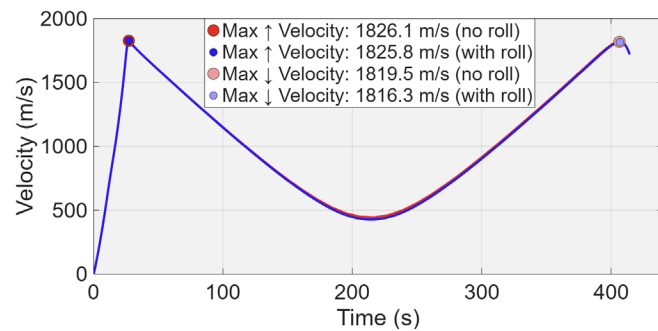


Figure 21. MATLAB 4DOF Simulator - velocity over time, comparing rolling and non-rolling conditions.

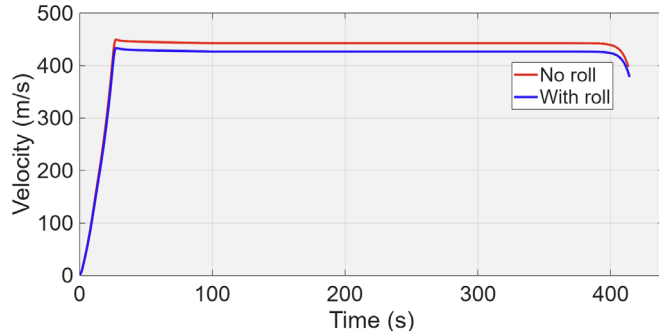


Figure 22. MATLAB 4DOF Simulator - horizontal velocity over time, comparing rolling and non-rolling conditions.

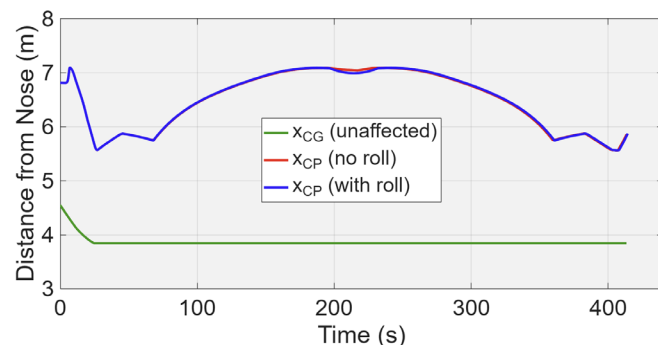


Figure 23. MATLAB 4DOF Simulator - centers of gravity and pressure over time, comparing rolling and non-rolling conditions.

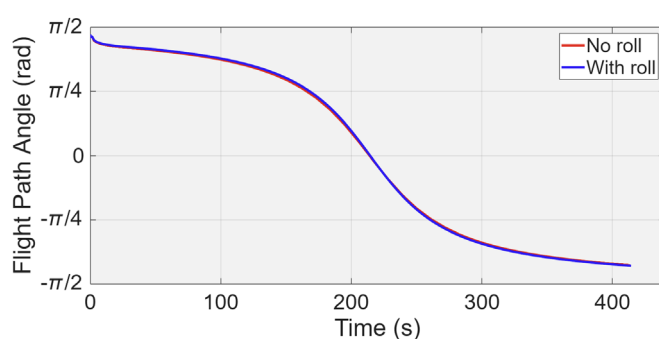


Figure 24. MATLAB 4DOF Simulator - flight path angle over time, comparing rolling and non-rolling conditions.

Results suggest that the Black Brant VC experiences a *greater* apogee, with the sacrifice of *smaller* distance under these rolling conditions. Roll rate will also *decrease* velocity due to energy being consumed during spin, however this energy helps to redistribute the already small differences in perturbing forces experienced about pitch and yaw

in a more even manner.

The direct effect roll rate has on passive stability of the Black Brant VC can be viewed with the figures below. A local simulation was run at time $t = 15$ in the same manner as the 3DOF stability subsystem, yielding many additional results considering yaw. Coning motion and coning decay are direct results of the 4DOF subsystem’s complex oscillator, displaying the tradeoff of taking longer to reach a steady state with less stress on the body frame.

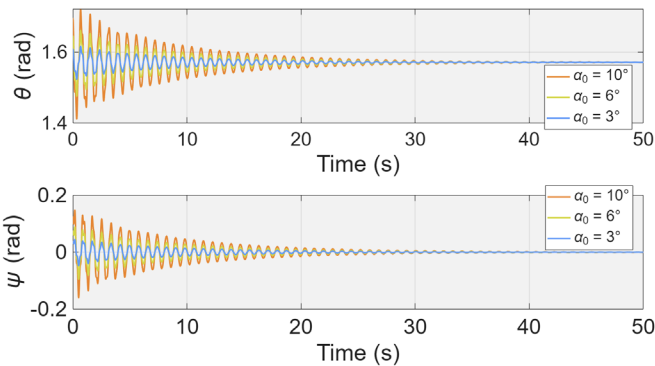


Figure 25. MATLAB 4DOF simulator - attitude timeplots.

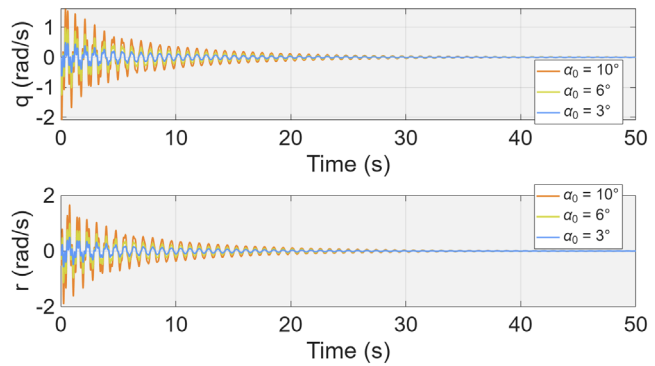


Figure 26. MATLAB 4DOF simulator - angular rate timeplots.

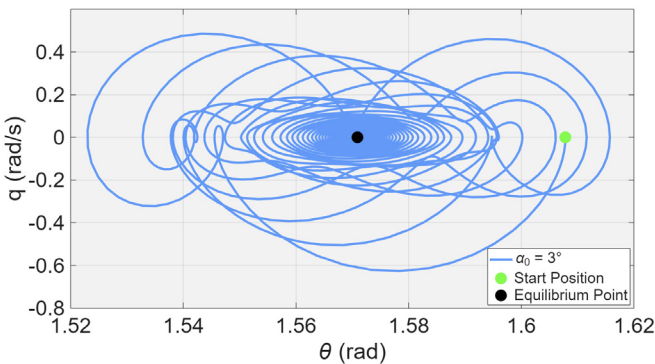


Figure 27. Pitch stability phase portrait, $\alpha_0 = 3^\circ$.

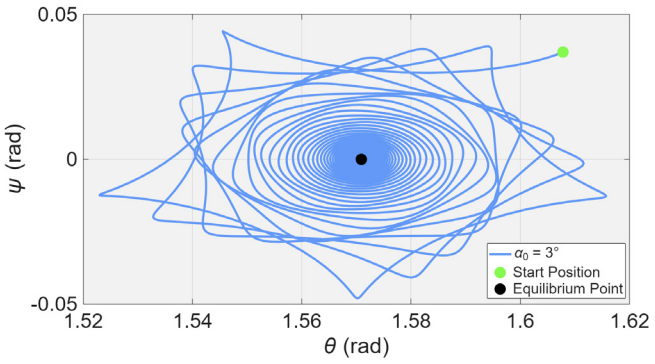


Figure 28. Coning motion phase portrait, $\alpha_0 = 3^\circ$.

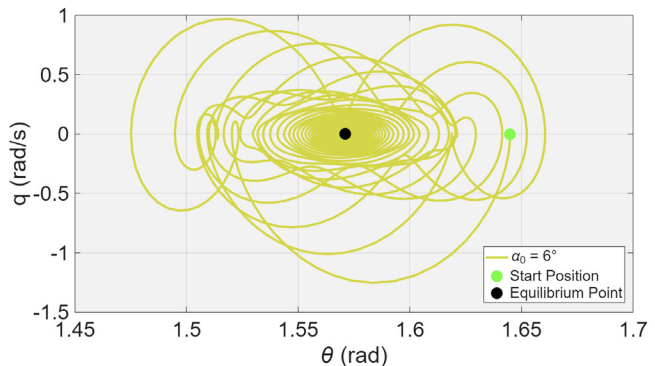


Figure 29. Pitch stability phase portrait, $\alpha_0 = 6^\circ$.

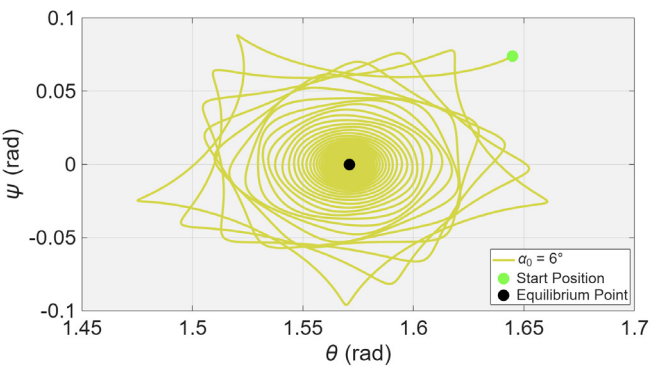


Figure 30. Coning motion phase portrait, $\alpha_0 = 6^\circ$.

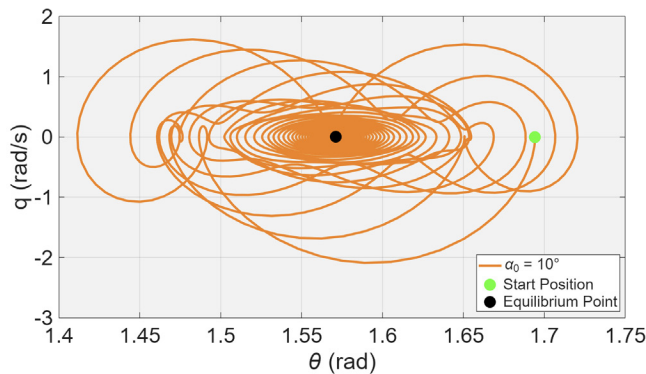


Figure 31. Pitch stability phase portrait, $\alpha_0 = 10^\circ$.

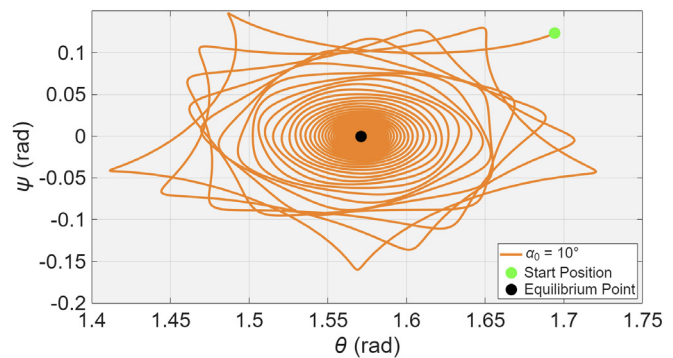


Figure 32. Coning motion phase portrait, $\alpha_0 = 10^\circ$.

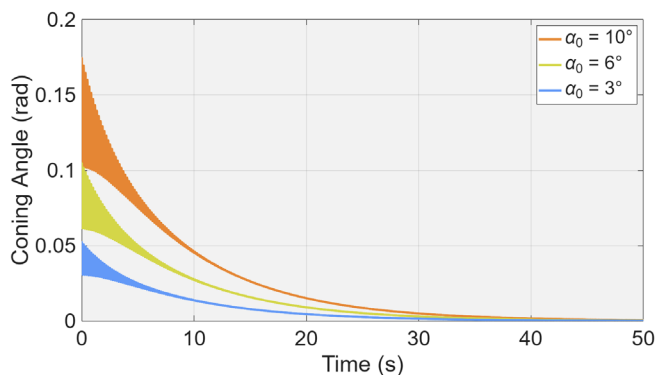


Figure 33. Exponential decay of coning motion.

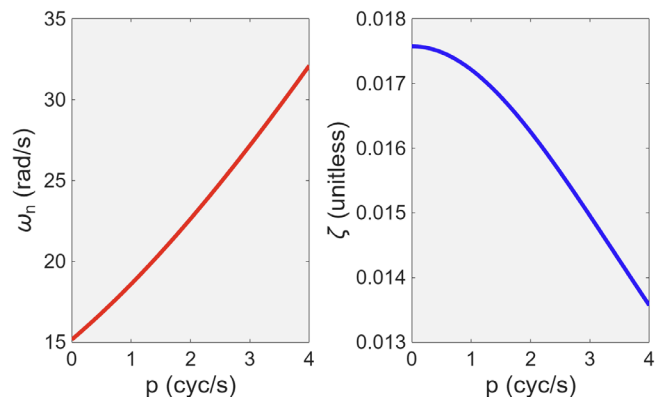


Figure 34. Natural frequency and damping ratio ζ calculations.

DISCUSSION

With the 2DOF model, rocket trajectory and stability can be mathematically coupled, as shown in **Equation 17**, but not without large physical constraints. Both simulators report a flight duration of approximately six minutes and fifty-two seconds, and while MATLAB trajectory attributes agree with estimations produced by RASAeroII up to two orders, pitch attitude θ remains constant at $\frac{\pi}{2}$ radians. Subsequently, its derivative, pitching rate, remains zero. This is why angle of attack is always zero, proved in **Lemma 1**. This results in an infinitely damped rocket - no amount of perturbing forces can effect stability of the rocket in this model, thus **Equation 17** is insufficient for passive stability analysis.

Adding a third degree of freedom results in a complete picture of both rocket trajectory and stability about pitch - RASAeroII reports an apogee of **682879ft** under vertical launch, and the 3DOF model reports apogee at **682883ft**, a difference of only *four feet*, resulting in much higher agreement with RASAeroII. Angle of attack α is no longer forced to be zero, thus **Equation 22** allows for deviation from flight path angle γ , and stability is properly analyzed about pitch axis y^B . This is the minimal mathematical model that bidirectionally couples trajectory with stability, as was proved in **Theorem 2**, and passive stability of a rocket can be determined by viewing the solution timeplot of $\theta(t)$ and $q(t)$ from **Figure 14**, showing damped oscillations over time.

To further this article's research into new territory, a fourth degree of freedom was added to the model. By the result obtained in **Theorem 4**, roll is uncoupled from pitch and yaw in an truly axisymmetric rocket, and thus can be treated as a parameter in **Equation 32**. The 4DOF stability subsystem from **Equation 33** contains four equations, and the complex oscillator from **Equation 35** describes many results such as nose coning motion, pitch stability, and yaw stability, with and without roll. From **Figure 34**, it's clear that an increase in roll rate results in an increase of natural frequency for the Black Brant VC specifically, and decreases its damping ratio. Too much

roll rate for this rocket can inflate natural frequency until damping becomes too small, resulting in a stable limit cycle, as opposed to reaching an equilibrium point. This is much more stressful on the rocket's mainframe, and therefore every rocket will have some ideal roll rate, including zero, in which force averaging provides the best positive effect on passive stability and stress on the rocket's mainframe.

A rocket with excessive damping or a low natural frequency cannot always be directly corrected through roll, however inducing roll can alter the coupled dynamics of the pitch / yaw stability subsystem, which can modify the damping ratio and natural frequency through energy exchange between these two axes. This is because perturbing forces cannot be increased or decreased without structurally changing the shape and size of a rocket, but can be redistributed more evenly between the pitch / roll plane and the yaw / roll plane, causing a phenomenon known as force averaging. The best representation of how roll affects stability is shown below for the Black Brant VC, comparing 3DOF to 4DOF pitch stability phase portraits.

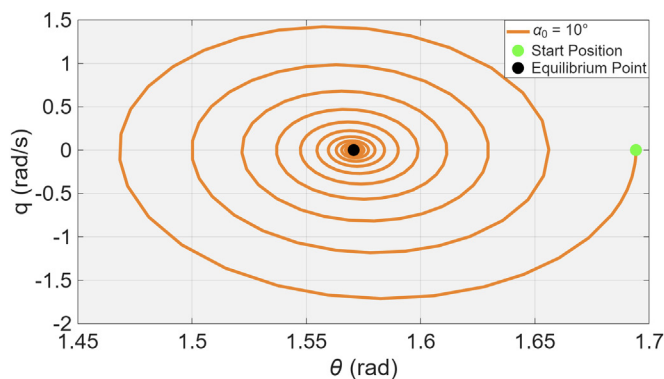


Figure 35. Pitch stability, 3DOF, no roll (inflated C_{m_q}).

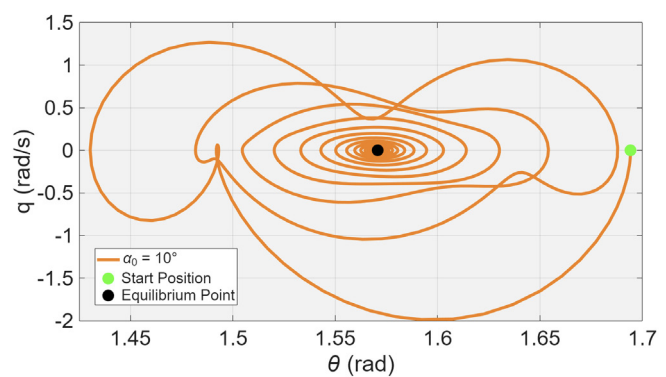


Figure 36. Pitch stability, 4DOF, with roll (inflated C_{m_q}).

By inflating the damping derivative fivefold, it's clear how force averaging in **Equation 34** eventually evens the pitch stability phase portrait in **Figure 36** to a spiral sink - the same spiral sink as the center of the 3DOF oscillator from **Equation 26**, as seen in **Figure 35**. By redistributing energy between these axes of rotation, roll seems to lessen the time spent at larger pitch attitude and pitching rate values, with the sacrifice of a slightly larger range in both respects.

Finally, the figures below highlight coning with pitching rate, and rates with pitch attitude - the yaw counterparts are similar in shape and size with different values. This unique and new insight into passive stability with roll allows this article to present exactly how a rocket is behaving within the pitch and yaw axes in local timespans.

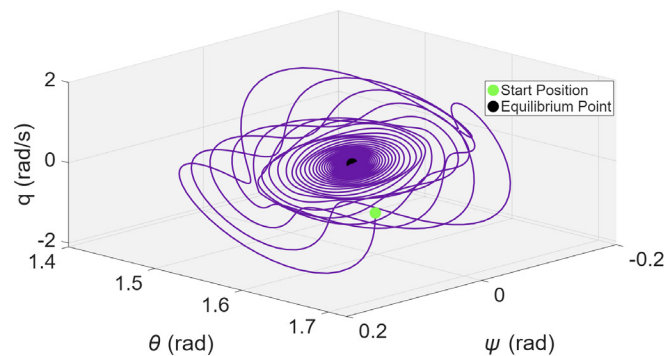


Figure 37. Coning motion with pitching rate.

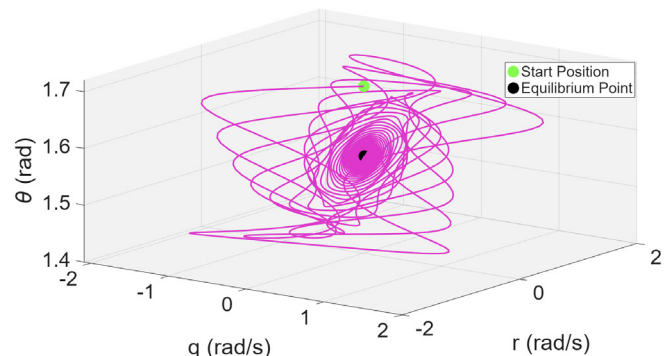


Figure 38. Angular rates with pitch attitude.

By considering the varying degrees of freedom covered in this article, **Table 1** is constructed, which highlights the level of passive stability considered by the oscillator produced by each model's stability subsystem.

Degrees of Freedom	Stability Oscillator	Results
2	$\theta'' + c\theta' = 0$	Damping only
3	$\theta'' + c\theta' + k(\theta - \gamma) = 0$	Stiffness and damping
4	$z'' + (c + i\Omega)z' + kz = 0$	Force averaging and coning

Table 1. Each oscillator derived and results of each model's stability subsystem.

CONCLUSIONS

The three degree of freedom model is necessary and sufficient for bidirectionally coupling rocket trajectory with stability. This model requires a minimum of six ordinary differential equations, and calculates copious amounts of data about a single rocket launch. Application of this model in MATLAB results in an accurate rocket simulator, justifying the massive leap in simplicity with physics and hand-solved solutions. The ability to track such results is expected of a 3DOF simulator, however the ability to view phase portraits of pitch stability at any snapshot in the rocket's travel is a special point of view unique to this article.

The four degree of freedom model has the potential to surpass rocket simulators currently available to the public in terms of simplicity and ease of understanding. Utilizing Euler's Equations of Motion to couple rotation about pitch with yaw is both novel and unique for this application - roll rate can not only be parameterized in such a model, but solved for as well. The important result of **Theorem 4** means rockets that may have had force biases in perturbation in other simulators, can be experimented with force averaging by adjusting roll rate in this article's 4DOF model.

Four degree of freedom rigid body models are most commonly used in vehicles with limited degrees of freedom, such as cars and boats. Applied to rockets, the axisymmetric condition removes the need to track two degrees of freedom each of horizontal translation and rotation, common for missiles and projectiles. A 5DOF model has since been implemented and coded in MATLAB by including translation along z^B , resulting in a three-dimensional picture of trajectory. Removing the axisymmetric assumption allows for a full rigid-body 6DOF model, which was completed in June of 2026, and a second research article has been written and submitted for publication consideration all about 6DOF considerations. Applications of these models can easily be applied to ballistics and other similarly-shaped flying bodies for smaller modeling examples.

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Jonathan Bartik graduated from the University of North Florida in Jacksonville with his Bachelors of Science in Applied Mathematics in May of 2026. He began his career as a Systems Modeling Engineer at Torch Technologies in August of 2026, supporting the Army DEVCOM AvMC program under the S3I directorate for rocket and missile software development. Jonathan enjoys spending his time modeling, coding, and learning, and his hobbies include automotive mechanics, spending time with friends, and attending mathematics and engineering related conferences. The MATLAB scripts for the 3DOF and 6DOF (not included in this article) rocket simulations can be found using the GitHub link here: <https://github.com/jbartik03/Rocket-Dynamics-Modeling-Portfolio>.

PRESS SUMMARY

What's the simplest way to understand how a rocket stays stable as it blasts off into space? In the world of aerospace engineering, state-of-the-art simulators use expensive math models to track every aspect of rocket motion. They're powerful, but cost time and power, and they're impossible to understand without high-end equipment and proper security clearance. This article starts with the simplest model possible, building layer by layer until the essential physics behind rocket stability reveal themselves. Finally, just enough detail is added to capture oscillations and include spin. Something incredible comes of this: a rocket's "wobble" can be understood as a beautifully structured spiral motion - a harmonic oscillator governed by aerodynamics and spin. This article proves you don't need a massive computer to simulate rocket stability. With the *right* minimal model, the physics behind rocket dynamics becomes clear, intuitive, and beautiful!