Particle Beam Waist Location in Plasma Wakefield Acceleration: Introduction and Background

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ABSTRACT

The role of beam waist location in interactions between a plasma and a particle beam is not yet fully understood. Nonlinear effects with the plasma make an analysis of such interactions difficult. Five simulations are presented in this report, with the waist location of a beam of ultra-relativistic electrons propagating through one meter of self-ionized lithium plasma. The simulation parameters are chosen to model the recent experiment 167 at the Stanford Linear Accelerator, relevant to the design of future plasma wakefield accelerating afterburners. It is found that beams focused near the point of entry into the plasma propagate further into the plasma and accelerate witness particles to a greater maximum energy before disintegrating. These results could indicate that ion channel formation is dependent on the drive beam waist location and that the plasma accelerating medium can have an observable effect on the focusing of the drive beam.

I. INTRODUCTION

Experiments using high energy particle accelerators have led to discoveries about the structure of matter, astrophysical processes, and the early history of the universe [1]. Future colliders that accelerate particles to higher energies could provide insight into the nature of mass and the unification of the fundamental forces of nature [2]. Constructing higher energy particle accelerators could contribute to research in many branches of physics. Particles accelerated to lower energy are useful for other purposes and are an important source of radiation for research and medical devices.

In conventional colliders, particles are accelerated using electric fields produced by microwave radiation. These fields travel synchronously with the particles to be accelerated [2]. The energy of the accelerated particles is a function of the strength of the accelerating field and the distance over which the field acts.

The fields that can be achieved by this radio frequency (RF) technology are fundamentally limited by the structural integrity of the collider apparatus. RF accelerating fields are generated in metal devices called slow wave cavities. Increasing the field in these cavities far beyond levels employed in current high energy colliders leads to electrical breakdown within the cavities and ionization of the materials with which the cavities are constructed [1].

Because the fields achievable using RF technology are limited, the energies that can be obtained with this technology have thus far been increased by increasing the distance over which the accelerating field acts. The highest energy RF collider is currently the Large Hadron Collider under construction at CERN [2] and scheduled to be completed in 2007. At this enormous facility, protons will circle a loop 8.7 km in radius before colliding with 7 TeV [Tera electron volts or 10^{12} eV] of energy. Continuing to advance the energy frontier of RF accelerators by increasing the size of the devices faces shortfalls of funding and resources. Plans for a Superconducting Super Collider that would be 28 km in diameter and cost $8 billion to construct were cancelled by the U.S. Congress in 1993, and a proposed linear collider 30 km in length has yet to receive funding [3].

New techniques for accelerating particles using plasmas could soon produce

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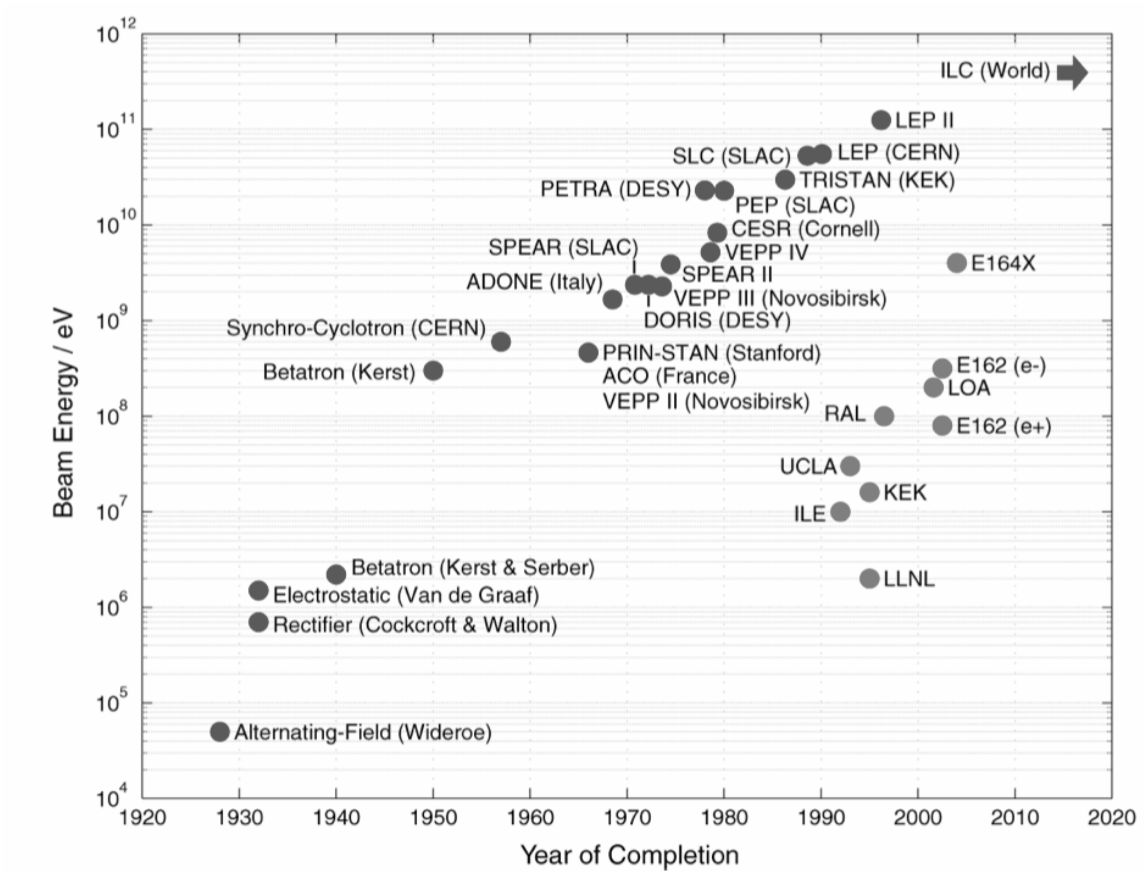


Figure 1. The Livingston plot showing the energies of RD (dark gray) and plasma (light gray) acceleration techniques. Energies obtainable in plasma acceleration experiments have risen rapidly over the last decade and may soon surpass energies achievable with RF technology [4].

smaller, less expensive colliders and extend the energy frontier beyond that set by current RF technology. In a plasma accelerator, an ionized gas of electrons and positive ions is used to generate accelerating fields. The fields that can be achieved in plasmas are over a thousand times greater than those that can be achieved in RF accelerators [3] and are not limited by the breakdown of the apparatus [1].

Energies achievable using plasma acceleration have been increasing by an order of magnitude every five years for the past decade [2], as shown in Figure 1. Due to larger accelerating fields, plasma acceleration devices are generally much smaller than comparable RF accelerators. Smaller accelerators require smaller beam sizes. Fortunately, a plasma can act as a focusing lens in certain situations.

While plasma particle accelerators may replace RF technology at the energy frontier, compact lower energy plasma accelerators could be used in the fields of materials science, structural biology, nuclear medicine, fusion research, food sterilization, and medical therapy. Plasma accelerators could also provide new sources of x-rays and gamma rays. Observing the behavior of charged particles in plasmas provides an opportunity to study the fields produced in this unique ionized environment and may contribute to the understanding of naturally occurring plasmas in astrophysical objects [1].

Plasma wakefield acceleration (PWFA) is a particular technique that shows promise for producing plasma accelerators at the energy frontier [4]. In PWFA schemes, a beam of ultra-relativistic[1] ($\gamma >> 10^3$) particles is injected into a plasma. The

[1] All symbols used in this report are defined in Table I.

behavior of the particles during acceleration and the maximum achievable energy is sensitive to the properties of the plasma and particle beam that is used.

The point at which the particle beam is focused is one parameter that affects the PWFA process. The impact of focusing distance on the interaction of a plasma and particle beam is not yet fully understood. The fields generated by the propagation of the particle beam through the plasma are highly nonlinear [5], and so an analytical description of the situation is difficult. A better understanding of the role of beam focusing distance in PWFA situations could lead to improved understanding of the physics of beam propagation within a plasma. The effects of beam focusing distance are also important to other areas of plasma research, such as inertial confinement fusion [6].

The envelope of a beam in an optical system generally focuses to a plane of minimum transverse spot size, known as the waist of the beam, and then expands in the transverse direction in a manner that is longitudinally symmetric [7], as shown in Figure 5.

This study presents the results of computer simulations performed with the QuickPIC algorithm [8] in which the location of the beam waist is varied relative to the location of the plane at which the particle beam first encounters the plasma. The waist location is adjusted in 5 cm increments from 0 to 25 cm in simulations of a 42 GeV [Giga electron volts or 10^9 eV] electron beam propagating through a plasma of ionized lithium.

I find that focusing the particle beam near the entrance of the beam into the plasma, with the beam waist positioned at 0 and 5 cm, results in a longer propagation distance for the particle beam and a higher maximum energy of the accelerated particles relative to beams focused more deeply into the plasma, such as a beam with 20 or 25 cm waist location. Although shallowly focused beams propagate a longer distance than those focused more deeply within the plasma, shallowly focused beams appear to be more susceptible to instabilities.

These simulations are directly relevant to the recent experiment 167 at the Stanford Linear Accelerator Center (SLAC). In this experiment, the SLAC electron beam was passed through a meter of lithium vapor that became ionized upon impact of the electron beam. Some beam particles achieved spectacular energy gain, doubling in energy from 42 GeV to over 80 GeV in a distance of 1 m [9]. However, the experimental results show a spectrum of particle energies with some particles with energies far below those seen in computer simulations. The study of beam waist position in PWFA suggests that these particles are likely self-trapped during the acceleration process.

Based on the simulations reported here, it appears that the minimum spot size of the beam in the E-167 simulations may not be as small as initially believed, which is contrary to the expectation that the plasma should act as a focusing device. These results may also challenge the assumption that the envelope of the beam in the plasma evolves as if the beam were propagating in a vacuum.

II. BACKGROUND: PLASMA WAKEFIELD ACCELERATION

a. Basic Wakefield Formation

In most plasma acceleration schemes, a high energy beam, called the

Parameter	Symbol
Electron mass	m
Speed of light in vacuum	c
Beam (plasma) number density	n_b (n_p)
Electron plasma frequency	$\omega_p = (n_p e^2/\varepsilon_0 m)^{1/2}$
Collisionless plasma skin depth	c/ω_p
Electron plasma wave number	$k_p = \omega_p/c = 2\pi/\lambda_p$
Total number of beam particles	N
Longitudinal r.m.s. beam spot size	σ_z
Transverse r.m.s. beam spot size	σ_r
Beam normalized velocity	$\beta = v_b/c$
Beam Lorentz factor	$\gamma = (1 - \beta)^{-1/2}$
Beam betatron oscillation frequency	$\omega_b = \omega_p/(2\gamma)^{1/2}$
Beam emittance	ε
Beam normalized emittance	$\varepsilon_N = \gamma\varepsilon$

Table 1. Definition of symbols used [10].

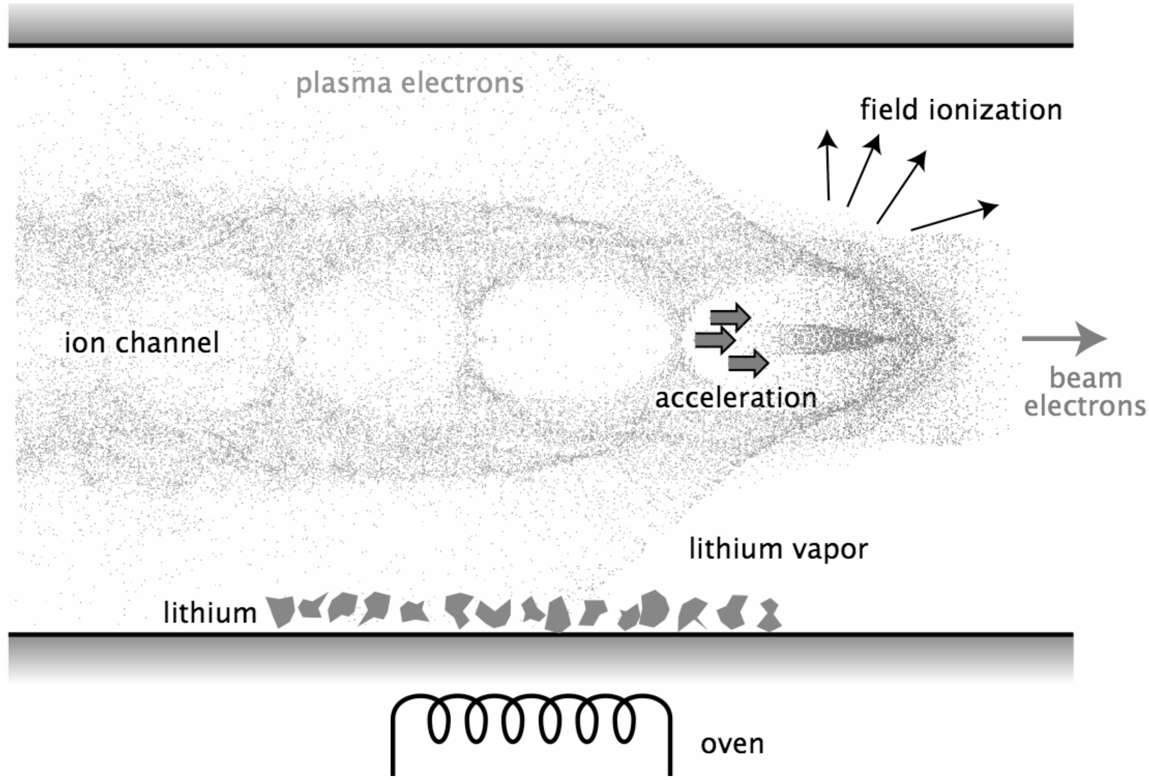


Figure 2. Wakefield formation—a drive pulse expels plasma electrons, creating a column of positive ions; plasma electrons oscillations create 'buckets' containing large electric fields that can be used for particle acceleration [4].

drive beam, consisting either of laser photons or charged particles, is shot into the plasma. Plasma wakefield acceleration (PWFA) refers to the case in which the drive beam is composed of ultra-relativistic charged particles ($\gamma >> 10^3$), usually electrons [10].

The plasma into which the drive beam is shot is initially electrically neutral. Upon impact from the drive beam, plasma electrons in the vicinity of the beam are expelled to preserve the charge neutrally of the plasma [10]. The positive ions are much more massive than the negative plasma ions and are approximately unaffected by the incoming drive beam. The plasma electrons beyond a few skin depths from the beam path are unaffected by the presence of the beam [5].

The behavior of the beam in the plasma depends on the relative densities of the plasma and the incoming drive beam. If $n_b < n_p$ and $k_p\sigma_r << 1$, meaning that the transverse beam spot size is much less than the plasma wavelength, the beam is pinched by its own magnetic field [10]. Otherwise, if $n_b > n_p$, called the underdense regime, the plasma electrons are expelled to a distance of several collisionless skin depths from the beam axis [5].

The positive ion channel left in the wake of the drive beam exerts an attractive Coulomb force on the expelled plasma electrons. The electrons return to the beam axis in about a plasma period, or in the frame of reference of the moving beam, one plasma wavelength behind the drive beam [5]. Due to their momentum, the electrons overshoot the beam axis and continue to oscillate in the transverse direction. If $n_b >> n_p$ and $k_p\sigma_r << 1$, nearly all plasma electrons are expelled by the drive beams. The transverse beam dynamics are simplified somewhat in the *blowout regime*, as the radical focusing force produced by the positive ion column is linear. In this case, the ion channel can be treated as a uniform cylinder of positive charge, and the field within the ion channel can be obtained from Gauss's law,

$$E_r = -\frac{1}{2}\frac{n_p e}{\varepsilon_0} r \qquad (1)$$

As the particle beam propagates forward, those oscillating electrons set up a wakefield consisting of a series of regions of positive charge, known as *buckets*, each followed by a dense region of negative charge, as shown in Figure 2. High electric fields can develop in the region with the positive ion channel at the rear of each bucket immediately preceding the region of dense negative charge.

Linear theory predicts that the peak accelerating field that can be obtained using the PWFA method is [10],

$$(eE)_{linear} = 240\,(MeV/m)\left(\frac{N}{4\times 10^{10}}\right)\left(\frac{0.6\,mm}{\sigma_z}\right)^2 . \qquad (2)$$

Although the underdense regime, $n_b > n_p$, is desirable for particle acceleration, linear theory is no longer valid in this regime. Due to nonlinearities, the underdense regime must be studied with particle-in-cell (PIC) computer simulations. One important linear theory result, namely that $(eE) \propto (\sigma_z)^{-2}$, appears to remain valid when the underdense plasma condition is satisfied [10].

Acceleration gradients achieved in the underdense regime can be greater than those predicted by linear theory. As shorter drive beam bunch lengths become possible, acceleration gradients of more than 40 GeV/m can be realized [10]. In comparison, the SLAC accelerates particles to approximately 40 GeV in energy over a distance of about 3.2 km.

Achieving high energies with plasma accelerators relies on the ability to inject charged particles into the region of maximum accelerating gradient at the rear of the first bucket. A pulse, called the *witness beam*, is launched behind the drive beam. The timing is such that the witness beam experiences the maximum accelerating gradient once the wakefield is formed by the drive beam. As the drive beam propagates through the plasma, the particles in the witness beam are influenced by the accelerating field and gain energy.

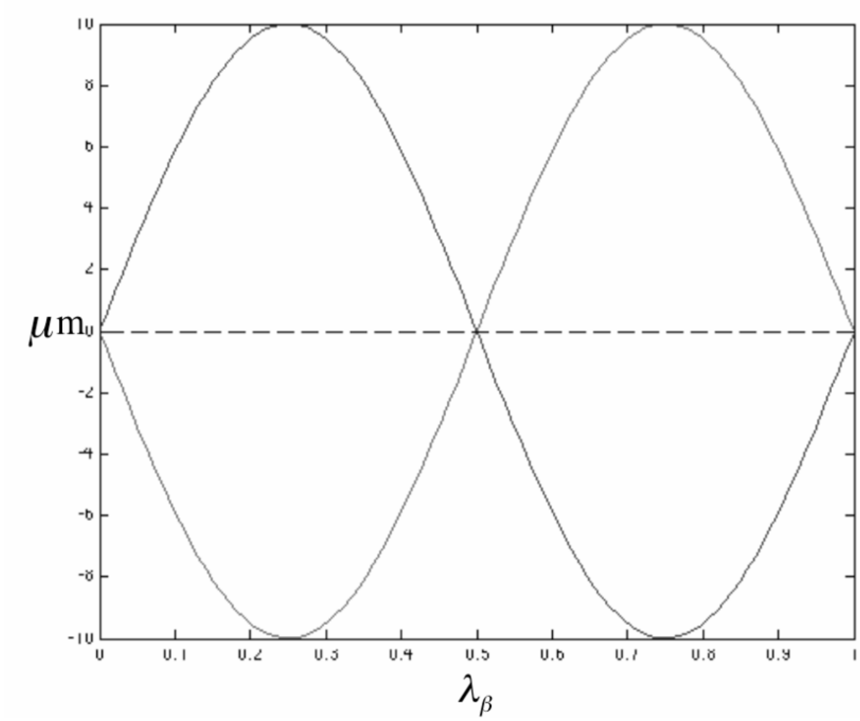


Figure 3. Two particles at the same energy undergo a single betatron oscillation. Because the particles are symmetrically located, the envelope of the beam experiences two complete oscillations.

Because the drive beam is ultra-relativistic, the witness beam must also be ultra-relativistic before injection. The witness beam travels at nearly the speed of light, and so most energy gained from acceleration is due to change in momentum rather than a change in speed. Thus the witness beam does not outrun the drive pulse as acceleration proceeds. In effect, the plasma acts as a transformer: the energy of the drive beam is transferred to the plasma in the formation of the wake, which in turn transfers energy to the witness beam.

b. Betatron Oscillation

The focusing force exerted by the ion channel can cause the electrons of the drive beam to oscillate about the axis of the beam. In the blowout regime, the focusing force exerted by the ion channel given by equation (1) can be recast in terms of the plasma wave number [11],

$$F_r = eE_r = -\frac{1}{2}\frac{n_p e^2}{\varepsilon_0} r = -\frac{1}{2} m c^2 \left(k_p\right)^2 r \quad (3)$$

The relativistic equation of motion for a single beam electron is thus,

$$F_r = \frac{dp_r}{dt} = \frac{d}{dt}(\gamma m v_r) \quad (4)$$

Because the beam is ultra-relativistic, the relation z = ct can be used to rephrase (4) in terms of the longitudinal location z,

$$\frac{d\left(\gamma m \frac{dr}{d(z/c)}\right)}{d(z/c)} = -\frac{1}{2} m c^2 (k_p)^2 r$$

or

$$\frac{d}{dz}\left(\gamma \frac{dr}{dz}\right) + \frac{1}{2}(k_p)^2 r = 0 \quad (5)$$

Equation (5) can be written as the equation of motion for a simple harmonic oscillator with the definition[2] $k_\beta = k_p/(2\gamma)^{1/2}$,

[2] The normalized emittance ε_N which appears in (6) is discussed in the later section “Phase space analysis of beam optics.”

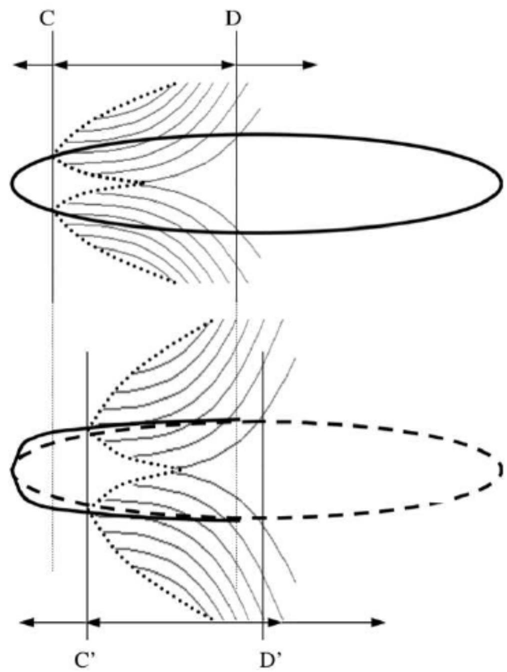


Figure 4. Self-ionization, in which the plasma medium is ionized and the wakefield is created by a single drive pulse. The ionization front, labeled D above, is the plane in which the entire transverse profile of the drive beam lies within the ion column. The portion of the beam ahead of the ion front is eroded, and the ionization front moves toward the rear of the drive pulse.

$$\frac{d}{dz}\left(\gamma \frac{dr}{dz}\right) + \gamma (k_\beta)^2 r = 0. \quad (6)$$

The transverse oscillation of beam electrons described by (6) is known as betatron oscillation. The frequency of this oscillation $\omega_\beta = \omega_p/(2\gamma)^{1/2}$ is energy dependent, and thus particles in a non-mono-energetic beam can undergo betatron oscillation at varying frequencies. Interference of differing betatron frequencies can lead to scalloping in the beam shape, as shown in Figure 17, which results as the wakefield acceleration process widens the energy distribution of the drive beam.

Betatron oscillation of individual particles within a beam can cause the envelope of the beam to oscillate. An equation describing the beam envelope can be obtained by summing the oscillations of all individual electrons. The result is [10],

$$\sigma_r''(z) + \left[\left(k_\beta\right)^2 - \frac{\left(\varepsilon_N\right)^2}{\gamma^2 \left(\sigma_r\right)^4} \right] \sigma_r(z) = 0 \,. \quad (7)$$

The frequency of envelope oscillation is twice that of the oscillation frequency of individual particles. This fact is a simple consequence of the geometry of the beam, as shown in Figure 3.

It is possible to eliminate the oscillation of the beam envelope by setting $\sigma_r''(z) = 0$ in (7). The initial radius that satisfies this condition is known as the matched radius $r_{bm} = \left(\varepsilon_N / \gamma k_p\right)^{1/2}$ [10]. If the radius of the beam at entry to the plasma is any other than the matched radius, the envelope of the beam will undergo betatron oscillations.

A particle beam with sufficiently high current density can simultaneously ionize a neutral gas into a plasma and create the wakefield structure. The radial electric field is that which governs the formation of the ion channel. The maximum radial electric field is proportional to the current density of the drive beam [12],

$$E_{r,\max} \propto \frac{N}{\sigma_r \, \sigma_z} \,. \quad (8)$$

Thus short beams with small spot size are most effective at producing self-ionized plasma.

As a first approximation, ionization of the plasma medium occurs when the electric field produced by the drive beam exceeds the electric force binding the plasma electrons to the positive ions in the un-ionized state. The ionization front is defined as the plane in which the entire transverse profile of the drive beam lies within the ion column, as shown in Figure 4.

The head of the drive pulse in the self-ionization process is ahead of the ion channel and thus does not experience the focusing force of the positive ions. The head of the beam expands, which by (8), causes the maximum radial electric field to decrease. As the field decreases, the ionization front moves further towards the rear of the drive pulse [12]. This process, known as *head erosion*, reduces the effective accelerating field within the wake and can lead to the eventual disintegration of the beam.

c. Plasma Focusing

The rate at which particles collide in an accelerator is as important to the usefulness of the apparatus as the total collision energy. Fewer particles are accelerated to the highest energies in a plasma accelerator relative to a comparable RF accelerator. To be competitive with RF accelerators at the energy frontier, plasma devices must achieve a large number of collisions of the highest energy particles.

One technique to ensure that an adequate number of high energy collisions occur is to reduce the transverse spot size of the beam while holding the beam density constant, thereby increasing the probability of collision and preserving the overall event rate. For plasma particle accelerators to be competitive at high energies using existing techniques, the particle drive beams must be focused down to a few tens of nanometers [3].

The plasma accelerating medium itself can act as a focusing lens, contributing to the desired reduction in spot size. The radial focusing force due to the ion channel in the blowout regime, as given by (3), is linearly proportional to the distance from the beam axis. This focusing mechanism can reduce beam spot size by a factor of about 2 to 4 using PWFA techniques [3].

d. Plasma Afterburners

PWFA technologies can be used to extend the energy range of existing RF accelerators. Devices called *plasma afterburners* are currently being developed that may, in the future, increase the energies of existing colliders by a factor or 2 or more.

Afterburners consist of a tube of plasma placed in the beam line of an RF accelerator near the collision point. Particles accelerated using RF fields serve as the drive pulse for the plasma afterburner. The particle bunch produced by the collider ideally consists of two micro-bunches, each approximately 100 fs [femto-seconds or 10^{-15} seconds] in length, separated by about 100 fs. The first bunch

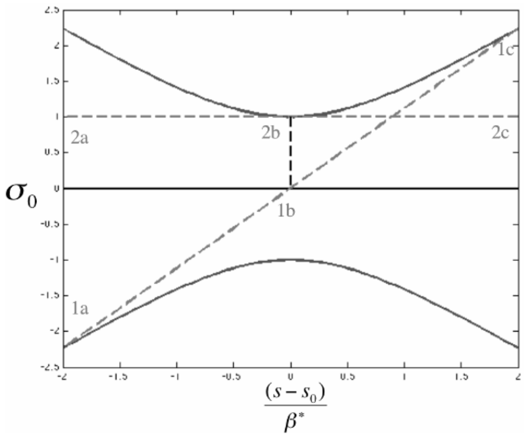


Figure 5. One-dimensional profile of the beam envelope (curved lines) near the waist (vertical dashed line). The thinner dashed lines track motion of two particles as the beam propagates in the longitudinal direction. The phase space evolution of these particles is shown in Figure 6.

ionizes the plasma and excites the wake, and the second bunch, containing about one-third as much charge as the drive pulse, is trapped in the wake and accelerated.

Current predictions imagine plasma columns about 10 m long, one on each side of the collision point of the collider. Due to the energy spreading that occurs in the plasma, it would be necessary to focus the beam to tighter spot sizes to increase the collision rate of the collider. Fortunately, this could be accomplished with plasma lenses placed immediately at the exit of each afterburner. Due to the different dynamics of wakefield formation by positive drive particles, the positron beam afterburner would have a hollow channel.

e. Present Difficulties

Radiation Damping—a beam undergoing betatron oscillations looses energy due to radiation resulting from the acceleration of the beam particles. This energy loss reduces the amount of energy available for acceleration. Betatron oscillation is essentially a simple harmonic oscillation process, and thus the magnitude of the acceleration experienced by the charge depends strongly on the initial displacement of the charge from the axis of the beam when the beam enters the plasma [13]. The resulting Radiative losses could lead to an undesirable spread in energy of the accelerated particles.

Although the energy lost to radiation is undesirable, for acceleration purposes, the resulting photons can be a useful source of radiation for optical purposes [4].

Hosing Instability—the hosing instability refers to the tendency of small perturbations of a beam propagating through a plasma to be amplified. The effect is due to a non-linear coupling of the beam electrons to the plasma electrons at the edge of the beam [14]. The non-linear growth of transverse perturbations can eventually lead to the breakup of the beam.

Experimental results and simulations show that the effect of the hosing instability is not as severe as initially thought [4]. Imperfections in the beam, such as asymmetries and longitudinal density gradients with the beam, tend to suppress the growth of hosing instabilities. However, the hosing instability can become significant when propagating PWFA beams distances on the order of a meter or more through a plasma.

Trapped particles—as a particle travels through a plasma, the resulting wakefield can trap plasma particles. Trapping can result from multiple ionization of the plasma medium.

Since particles can become trapped at any point and are rarely trapped in the region of highest field gradient, the energy gain of the trapped particles is usually much less than that of the witness bunch. Hence these trapped particles are doubly problematic in that they appear as low energy noise in energy diagnostics and also extract energy from the wakefield that could otherwise go to acceleration of the witness bunch.

III. BACKGROUND: EMITTANCE TWIST PARAMETERS

a. Beam Focusing

Often it is desirable to reduce the spot size of a beam used in an optical system. This can be accomplished by imparting the beam particles farthest from the beam axis with transverse momentum towards the beams axis. If the beam particles receive no further perturbations, the beam envelope contracts to minimum spot size. The location of the plane at which this minimum occurs is known as the *beam waist*. The beam envelope expands symmetrically on the opposite side of the beam waist [7], as shown in Figure 5.

In the case that the beam is not azimuthally symmetrical, the waist location can vary for differing transverse directions. The beam envelope will evolve independently in the two planes formed by the transverse coordinate axes, respectively, and the beam axis.

Consider a single transverse dimension of a beam undergoing focusing. Let this arbitrarily-chosen direction be called *y*. Beam particles farthest from the beam axis are given transverse momentum to reduce the spot size of the beam. Hence particles at large –*y* receive momentum in the positive direction and particles at large +*y* receive momentum in the negative transverse direction.

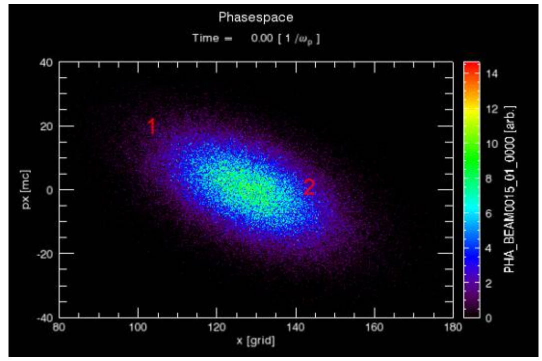
The behavior of a representative particle is shown in Figure 5, labeled ‘1’. Assuming that no forces act on the beam particles, the transverse momentum of particle 1 carries it through the beam axis. The particle continues past the beam axis. Similar behavior for other beam particles results in expansion of the beam envelope as the beam propagates past the plane of the waist.

Because of focusing imperfections, some beam particles off the beam axis receive no transverse momentum. These particles limit the minimum spot size of the beam. A representative particle is labeled as ‘2’ in Figure 5. As the beam propagates, such particles maintain constant transverse positions.

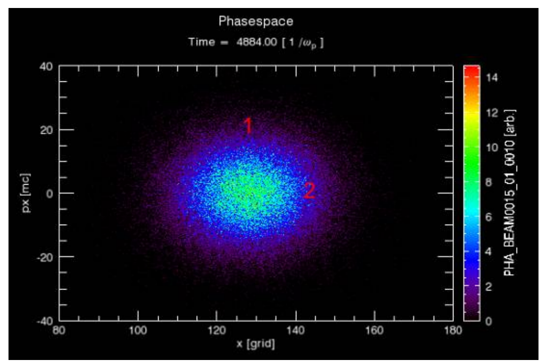
b. Phase Space Analysis

It is useful to analyze the focusing of such a beam in phase space. The phase space description leads to the definition of emittance and the Twist parameters. The emittance is an important characterization of the angular spread of the beam, and the Twist parameters can be used to specify the waist location of the beam.

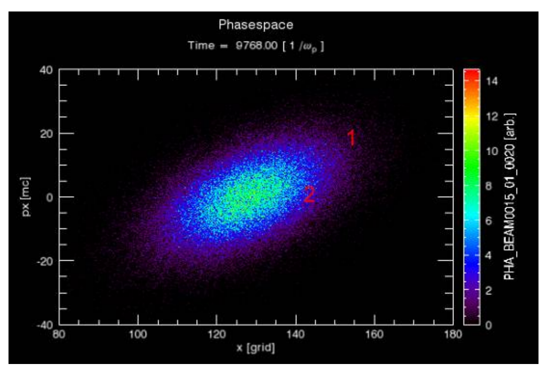
The phase space evolution of a beam undergoing focusing, including the representative particles 1 and 2 referenced in Figure 5, appears in Figure 6. Because the beam is assumed to be propagating in free space, the momentum of any given beam particle is unchanged. Hence the vertical position of any point in the phase space diagram is fixed.



(a) Converging beam approaching waist.



(b) Beam at waist



(c) Diverging beam located at point symmetric about waist in transverse direction.

Figure 6. Phase space evolution in one spatial dimension of a typical optical beam near waist. Red numbers show locations of two particles whose trajectories are plotted in Figure 5.

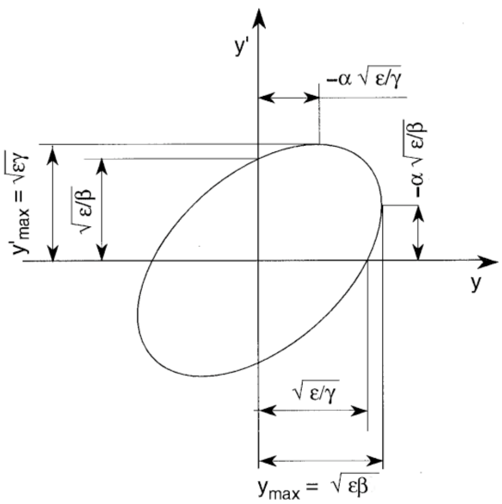


Figure 7. Geometrical definition of emittance and Twiss parameters on a theoretical beam of definite envelope and constant density.

However, the transverse position of beam particles can change as the beam propagates. The change in transverse position is proportional to transverse momentum, which is represented on the vertical axis of the phase space plot. Hence the horizontal movement of a point in phase space is in proportion to its constant vertical position.

As stated above, particles initially with the most extreme negative positions begin with the most positive momentum, and vice versa for particles at positive transverse positions. The anti-correlation of position and momentum results in an ellipse in phase space that slopes from the upper left to the lower right, as shown in Figure 6.

The horizontal axis of the ellipse corresponds to zero transverse momentum. Hence the particles with no transverse momentum, such as particle 2, which determine the minimum spot size, also determine the constant equatorial width of the phase space ellipse.

Let's consider a beam undergoing focusing prior to the beam reaching its waist. As the beam propagates, the spread in transverse position of the beam particles decreases. Particles with transverse momentum move towards the line y = 0 in phase space. Because the horizontal movement of points in phase space is in proportion to their vertical position, the phase space ellipse shears, with the portion in the upper half plane moving to the right and points below the horizontal axis moving to the left. At the waist, particles with maximal momentum magnitudes cross the beam axis, and so the vertical maxima of the ellipse are aligned so that the figure becomes circular.

As the beam propagates past the plane of the waist, the shearing of the phase space ellipse due to the initial transverse momentum of the beam particles continues. As particles with positive transverse momenta cross the beam axis and continue in the positive transverse direction, transverse position and momentum become positively correlated. The phase space diagram evolves to become an ellipse that is slanted from the lower left to the upper right, as shown in Figure 6.

c. Emittance

Liouville's Theorem is a mathematical statement that the area in phase space of an ensemble of initial conditions of a non-dissipative system is constant. This theorem can be applied to a beam propagating in vacuum, assuming that the beam has a discrete envelope in phase space so that the concept of area is well defined. In this case, it is meaningful to define emittance as a measure of the constant area of the phase space ellipse,

$$\varepsilon = \frac{area\ in\ phase\ space}{\pi} \tag{9}$$

In the phase space plot, transverse position is measured in units of length, whereas momentum is usually written using normalized units so as to be effectively unitless. The unitless momentum can be assigned a unit of radians, so that emittance is often expressed in units of length·radians.

The emittance is a measure of the initial transverse momenta present in a beam. With knowledge of the distance over which the beam is propagated, the emittance can give information about the spot size of the beam. Alternatively, emittance can be interpreted independently of the propagation distance as a measure of the angular spread of the beam. The normalized emittance $\varepsilon_N = \gamma\,\varepsilon$ takes into account the energy of the beam.

A transverse profile in phase space of the physical particle beam is not bounded by a discrete beam envelope. In this case, the interpretation of ε as the phase space area spanned by all of the beam particles is no longer exact. However, the emittance can still be defined in this case to represent an average beam size in phase space,

$$\varepsilon = \sigma_r \cdot \frac{v_T}{c} \qquad (10)$$

Where σ_r is the transverse r.m.s. [root mean squared] beam spot size at the waist and v_T is the r.m.s. transverse velocity.

d. Twiss Parameters

Let us return now to the discussion of a theoretical beam with discrete phase space envelope. Since the phase space ellipse of the beam is symmetric and has constant area, it can be uniquely characterized by three parameters [7]. The first of these parameter is the emittance. The other two parameters describe the shape of the phase space ellipse and are functions of the beam propagation distance *z*. These two functions are the Twiss parameters.

The amplitude function *β(z)* is defined to describe the point on the ellipse with maximal transverse displacement,

$$y_{\max} = \sqrt{\varepsilon\,\beta(z)} \qquad (11)$$

The unit of radians is dropped from *ε* in (11), and so *β(z)* is commonly expressed in units of length.

The other Twiss parameter *α(z)* is related to the point of maximal transverse momentum represented on the phase space ellipse,

$$y'_{\max} = \sqrt{\varepsilon}\sqrt{\frac{1+\alpha^2}{\beta}} \qquad (12)$$

Because *ε* and *β* carry the same units, modulo a dimensionless factor of radians, *α* is taken to be unitless.

Equation (12) can be written more conveniently by defining the quantity[3] $\gamma = (1+\alpha^2)/\beta$. With this definition, (12) becomes,

$$y'_{\max} = \sqrt{\varepsilon\,\gamma} \qquad (13)$$

Because the beam is assumed to be propagating in vacuum, the vertical position of each point of the phase space diagram is fixed. Hence *γ* is independent of the beam propagation distance. However, *α(z)* is dependent the propagation distance through *β(z)*. This geometric interpretation of the Twiss parameters is shown in Figure 7.

e. Relation of Twiss parameters to the Beam Waist

In the simulations described in this paper, the drive beam is assumed to focus as if propagating in a vacuum. This assumption is based on the fact that the plasma density is much less than the beam density. The focusing of the beam is modeled using the phase space analysis described above. The beam waist location is determined by specifying the values of the Twiss parameters at the plane at which the beam enters the plasma.

As noted above, the phase space plot of the beam becomes circular at the beam waist. From Figure 7, the beam waist is thus characterized by the point at which *α* = 0. The longitudinal position of the plane represented on a particular phase space diagram is related to the value of *α* at that longitudinal point [7].

$$z_{waist} - z = \frac{\alpha}{\gamma} \qquad (14)$$

Hence α > 0 implies that the beam is converging towards the waist, whereas α < 0 implies that the beam has passed the waist and is diverging.

The value of *β(z)* at the beam waist is a constant that parameterizes the minimum spot size of the beam. This value of *β* at the waist is written as *β**.

[3] The γ and β defined as Twiss parameters bear no relation to those in the relativistic factor $\gamma = (1-\beta)^{-1/2}$.

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Editorial Note: *The research outlined above will be continued in the next issue (June 2007), where Methods, Data and Discussion sections will be reported.*

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